ISSN 2045-3345

ISSN 2045-3345

RISK IN BANKING ACTIVITIES AND ITS QUANTITATIVE DETERMINATION

Irina Maria Dragan[I]
Bucharest University of Economic Studies, Romania

ABSTRACT

The volatility becomes a constantly characteristic of the present economic environment. Considering those circumstances, the decisions are affected by risks in an expanding capacity. Since the management decisions are typically taken on the basis of incomplete information, based on some partial data and in conditions of uncertainty, these are associated with errors. Because the risk is regarded as a probability, therefore it is worth to investigate the nature of what is called the statistical risk. It plays an important role in the framework of statistical inference. The statistical feature of the decisional area determines that the error's probabilistic measurement to be made by the risks. In this paper there are depicted and adjusted the risks, such as Type I, Type II and Type III. The variance resulting from measurement error becomes noise and thus it could decrease the power level. To be more specific, since a high degree of measurement error hinders the condition of the variable from being correctly indicated, it drags down the possibility of correctly detecting the effect under study. Those risk types are affecting the alternative hypotheses of the decision making processes and also the Taguchi risk and the potential index of a process.

JEL CLASSIFICATION & KEYWORDS

■ C44 ■ D81 ■ STATISTICAL RISK ■ RISK OF ERRORS ■ DECISIONAL PROCESS

INTRODUCTION

One of the simplest decisional problems under uncertainty conditions is to accept or to reject a statistical hypothesis, hypothesis that can be true or false. Uncertainty in the decisional process occurs due to sampling: we work with a part (or with parts of the origin population), consequently, with samples of a certain "whole" (set, batch, population etc.) and not with the whole collectivity, the decision to accept/reject being made on the basis of examining only that part represented by the sample. The error is the wrong decision and the probabilistic measure of making the error is the risk. In this paper was developed the subject of uncertainty of decision and of risk in general, and particularly, of the statistical one.

Uncertainty and management decision

The usual definition of uncertainty says that we are dealing with something uncertain, undetermined and doubtful: uncertainty is a doubt about something or somebody. Certainty is considered as the opposite of uncertainty and signifies the fact that a certain fact, event, situation will take place or is taking place under our sights. We mention among the uncertainty sources: incomplete, partial information on a certain entity, lack of information, inadequate rendition of information, wrong assignment of causality.

Malita and Zidaroiu have published a work where they present the mathematical formalization of the decisional process and of the decision – making process (in the economic, medical, technological, administrative etc. field) having as theoretical fundamentals – among others – the probabilistic – statistical procedure and that one related to the utility theory. If we are interested of specific fields, as, for instance, the metrology: we are dealing with the measuring uncertainty, that, according to "International Vocabulary of Basic and General Terms in Metrology" 1992 is a parameter associated to the result of a measuring which characterize the dispersion of values (this parameter can be a standard deviation, a certain interval de statistical coverage). Here, uncertainty gets a correct contour by a statistical indicator, by formulae, expressing a certain fact – namely: how far or how close we are to the real value of the physical magnitude subjected to the measuring process (Hopkin, 2012).

We do not have, in mathematical statistics, a definition of the idea of uncertainty of a certain system: entropy, negentropy, information, informational energy etc. provide an image of the respective system state as against an eventual "standard system" and obviously in relation to the known subject, to the anthrop-social element. Such, uncertainty appears as the fruit of human ignorance, and its manifestation form is the variability, which if it is not controlled, can generate what we usually call a risk: namely, to make a wrong decision into a given situation, where the required information is distorted because of this exaggerated variability. Barsan-Pipu and Popescu in a profile work, consider that risk is however an uncertain event, which can very well not to occur. Uncertainty is of two types: ontological and cognitive one. The ontological uncertainty (or the nondeterministic one) deals with problem, abandoning the idea that the future states come from current states. The calculation using such probabilistic knowledge will lead to choosing a solution, which might not be the best solution, but, for sure, the choice was made because the solution has the highest probability to be the best solution. Anyway, the selected solution could be a worse solution, but it was a right, rational selection, with the highest probability to be the best solution. The probabilistic thinking is the way of being rational and of making the right decisions into a partially nondeterministic world. The cognitive uncertainty expresses the limits of our knowledge, thus, the decision – maker assigns manageability to different events. But, unlike the uncertainty, the risk is characterized by the possibility to be quantified by probability laws. Consequently, a discussion about the concept of risk is required.

Uncertainty and the connection with the decision under risk

The risk is a phenomenon which affects and influences any human action. This can be defined as a commitment bearing an uncertainty, due to the probability of money – making or money - losing. The social, economic and cultural progress would not have been possible without assuming certain risks in decision – making, both at personal level and within

[I] irina.dragan@csie.ase.ro

RESEARCH JOURNAL OF ECONOMICS, BUSINESS AND ICT

organizations. A rigorous definition of risk says that "the context where an event occurs with a certain probability or where the extension of the event follows a probability distribution"[1]. This definition outlines that the actions taken at a given moment by a person or by an organization, lead to an event or to a set of events, which a probability or a distribution of occurrence probabilities are associated to and the risk is nothing else but what results from the combination of the occurrence probabilities and their consequences. Etymologically, the word comes from the Latin re-secare - which means a balance breaking[2] (in French: risqué). Further on, we present some definitions (dictionary) related to this rendition.

Mic Dictionar Enciclopedic (Stiintifica and Enciclopedica Publishing, Bucuresti, page. 809): "Risk: danger, possible inconvenience". Then, it follows: the "contractual risk", i.e.: the debtor must bear the injury consequences of releasing the creditor from the obligations he had to him, as a result of non-performing, by debtor, of his duties from causes which are not ascribed to him. Here, we meet also the notion of working risk – the owner shall bear the damages resulting from the loss or from the destruction of work by force majeure. We shall also mention the notion of insured risk. An immediate derivative is risky, i.e.: full of risks, dangers exposed, unsafe etc.

Merriam – Webster's Collegiate Dictionary (10th Edition 1994, Springfield, Mass., pp. 1011) providing historical details, i.e.: when "risk" term (and its derivatives) started to be used in English: the possibility that something bad or unpleasant (such as an injury or a loss) will happen; someone or something that may cause something bad or unpleasant to happen – see also Security Risk; a person or a think that someone judges to be a good or bad choice, for instance a loan etc. Larousse de Base (Dict. d'apprentisage du français), Libr. Larousse Paris, page 723): "Risk" is a danger that can be foreseen; an (abstract) disadvantage we are exposed to; an action involving risks is called risky or hazardous (hasardeuse).

Terry Lopez states may be the most concise but still comprehensive definition: "risk means danger to suffer a loss". The change of this image comes firstly from the introduction of the uncertainty concept into the rational decision model. Th. Craiovenu remarks the fact that "when uncertainty reaches high levels, it must be diminished, but the most undesirable level of uncertainty in life is that of full certainty.

Uyemura and Deventa (1993) present the idea that risk is the volatility (the variance) of the net cash flow for a business unit. By business unit, for the particular case of a bank, may be taken into account an agency, a branch or the whole bank.

Some conclusions appear clearly from all these definitions: the risk association to human factor ("the cars do not have troubles" and the lions of African savannahs "do not suffer damages" but "misfires" in their chase after impalas); the risk is indissolubly related to uncertainty or loss. The risk can or can not occur itself through the so-called risk factors, which, when they effectively operate, can cause loses of diverse nature and magnitudes. We have also to mention that despite the fact that there is a risk, in a certain situation, this is possible not to occur (the decision-maker is acting "on chance" and he is fortunate enough not to suffer losses, damages etc.). The authors' opinion is that risk is present in all human activities and for this reason the risk theory is a wide and difficult field of research. The mathematical modeling of risk has as start point the assumption that risk can be assimilated to the possibility of suffering a loss. As this possibility can be quantitatively expressed by probability, risk appears as a probability occurrence function of an undesired phenomenon, and also the presence of the adverse effects of this event.

Statistical indicators for risk assessment

Statistics – according to its numerous definitions, covers this too: "the science of decision-making under uncertainty (Lovric, 2011; Lindley, 2000). Statistical hypotheses are statements on one or several parameters, or on some distributions which are going to be validated by statistical tests. The decision on null hypothesis is made on the basis of statistical test algorithms. The statistical test being itself built with random elements, the decision-making bears an error risk. The null hypothesis (H_0) refers to statements to be subjected to testing, while the alternative hypothesis (H_1) refers to statements which will be accepted if the null hypothesis is rejected. Testing the hypothesis that the mean μ of a random variable X of a population is not lower than a given value μ_0:H_0 ($\mu \geq \mu_0$) and H_1 ($\mu < \mu_0$). Testing the hypothesis that the proportions of unconformable individuals of two populations p_1 and p_2 have the same (unspecified) value: H_0 ($p_1 = p_2$) and H_1 ($p_1 \neq p_2$). Testing the hypothesis that a random variable X has a normal distribution (with unspecified parameters). The alternative hypothesis: the distribution is not normal.

The critical region R_c is the region R_c of rejecting the statistical hypothesis and is called the critical region of the Euclidean space R_n. In order to build the critical region R_c of a test, there are usually used certain sample indicators of the form E (x_1, …, x_n) and the probability α to make a type I error. The critical region is defined by set R_c that satisfies the condition as the type I error probability $E \in R_c$, when hypothesis h is true, to be equal to α, i.e.: $P[E(x_1, x_2,\ldots, x_n) \in R_c;H] = \alpha$. Type II error probability is defined by relation: $P[E(x_1, x_2,\ldots, x_n) \in R^*_c \text{ nonH}] = ß$ where "non H" is the negation of hypothesis H. The critical regions are determined so that, if the null hypothesis is the true probability of rejecting the null hypothesis, it must be at most equal to a given value α, generally a low one (for example: 5% or 1%). Type I error probability. The value of this probability is always lower or equal to the test significance level. In certain situation, it is also called: "type I risk". Type II error probability. Its value depends on the real situation and can be calculated only if the alternative hypothesis is enough specified. Sometimes, it is also called: "type II risk". Power of a test. This probability, generally denoted by $\Pi_t = (1 – ß)$, corresponds to the null hypothesis, when it is false. We can also recall the fact present in literature, on a proposal, who introduces also a Type III error, that, according to his opinion, consists in solving false problems. We consider that a better clarification is required: what is, in fact, a "false" problem? Is the problem stated incorrectly or simply without any connection with the decision – maker's desire? The author does not clarify this aspect.

Risk manifestation in the management processes

From a statistical point of view, uncertainty and implicitly risk are modeled with the aid of some random variables. In this way, risk is considered a random (continuous) variable $X:\Omega \rightarrow R$ where (Ω, K, P) is a probability area. If $V(\Omega)$ is the set of all variables defined on Ω, and $M \subset V(\Omega)$ is a subset of $V(\Omega)$, then, a function $r:M \rightarrow R$ is a so-called measure

[1] Pierce, D. (edit) Macmillan Dictionary of Modern Economy, Codecs Publishing, page 353, 2000

[2] http://www.oed.com/

r(X) of risk X. Some examples related to the meaning of X: net profit of an investment, the loss suffered by an employee into a certain conjuncture, exceeding the alarm quota of a pollutant concentration etc. (Vose, 2008; Voda, 2009). Consequently, if X is the investment profit, the associated risk is:

$$r_1(X) = Var(X) = E[(X - m)^2] \quad (1)$$

where m = E(X) is the mean value of variable X. At the same time,

$$r2(X) = E[(T\text{-}X)_+^2] \quad (2)$$

Where T is the a so-called "failure limit" (a name given by Roy[3]), the investor wishes to have always a profit above the threshold T). If X is rendered as getting beyond the pollutant concentration with respect to a certain maximum level T, then, the polluting risk is:

$$r_3(X) = E[(X - T)_+^p], \quad p \in \{1,2,...\} \quad (3)$$

The used notation derives from "Lower Partial Moment" indicator of α order with respect to the limit T:

$$LPM_\alpha(T, X) = E[(T - X)_+^\alpha] = \int_{-\infty}^{T} (T - x)^p dF_x(x) \quad (4)$$

where $F_X(x)$ is a distribution function of X. If T is even m = E(X), and α = 2, then, the so-called semi variance is obtained:

$$SV(X) = E[(m - X)_+^2] \quad (5)$$

As can be noticed, we have to do with risk models using dispersion ($r_1(X)$) or dispersion with respect to a given value T. Further on, we shall present some variants of modeling the risk in processes management, mainly, for monitoring their capacity (Wright, 1995).

Taguchi risk. In fact, the forms $r_2(X)$ and $r_3(X)$, (relations 2, 3) are nothing else but the average risks associated to a quadratic form $L(x,T) = k(x - T)^2$ used by C.F. Gauss since 1809 (Kackar, 1985) and revitalized by Genichi Taguchi in "quality context": T is the target value of the measurable characteristic X, and L(x,T) is the loss quality function. Then:

$$R_T(x) = E[L(x,T)] = \int_D L(x,T) f(x) dx \quad (6)$$

is the Taguchi risk, i.e.: the mean value of variable L(x,T), where $f(x) = F'_x(x)$ and D is the definition field of X (usually $[0,+\infty)$). Genichi Taguchi revitalized Gauss quadratic function $f_{(x)} = a(x - x_0)^2$, $a > 0$, $x \geq x_0 \geq 0$ and associated it to the so-called quality loss:

$$L(x_0;T) = k(x_0;T)^2, \quad k > 0, x_0, T \in R \quad (7)$$

where x_0 is the measured value of the quality characteristic (X) and (T) is its target value (k is a constant depending on the specific case at hand).

If $f(x;\theta)$ is the density of X ($x \in D \leq R$, $\theta \in R$) then the average value

$$E[L(x;T)] = \int_D L(x;T) f(x;\theta) dx \quad (8)$$

is called Taguchi type risk (Kackar, 1985). Taking into account (7) we may write (8) as

$$E[L(x;T)] = k[Var(x) + (E(x) - T)^2] \quad (9)$$

and if X is normally distributed $N(\mu;\delta^2)$, we have:

$$E[L(x;T)] = k[\delta^2 + (\mu - T)^2] \quad (10)$$

[3] Roy, A. "Safety First and the Holding of Assets", Econometrica, 20(3): 431-449, 1952

The empirical risk (denoted by $\hat{R}_T(x)$) is therefore:

$$\hat{R}_T(x) = k[s^2 + (\bar{x} - T)^2] \quad (11)$$

where $\bar{x}$ and s are the well-known sample statistics. There is a straight forward link between Taguchi's risk and his own process capability index $\hat{C}_{pm}$ (Chan et al, 1988):

$$\hat{C}_{pm} = \frac{USL - LSL}{6 * \sqrt{s^2 + (\bar{x} - T)^2}} \quad (12)$$

where USL = Upper Specified Limit and LSL = Lower Specified Limit of the given quality characteristic $X \sim N(\mu,\delta^2)$ with T as its target value. We may write hence immediately:

$$\hat{R}_T(x) = k\left|\frac{USL - LSL}{6} * \frac{1}{\hat{C}_{pm}}\right|^2 \quad (13)$$

If USL – LSL = 6s - that is the minimal level for admissible process capability, we get:

$$\hat{R}_T(x;T) = k\left|\frac{s}{\hat{C}_{pm}}\right|^2 \quad (14)$$

and we draw the conclusion that the Taguchi's risk can be regarded as a function of the length of the specified interval USL - LSL measured in standard deviations units.

The theoretical Taguchi risk corresponding to (14) is:

$$\hat{R}_T(x) = k\left|\frac{\delta}{\hat{C}_{pm}}\right|^2 = \frac{k\delta^2}{\hat{C}_{pm}^2} \quad (15)$$

Denoting $k\delta^2 = M$ and $C_{pm} = X$, we shall have a hyperbolic dependence of the type $R = M/X^2$. If in C_{pm}, the true mean-value μ is just the target T, then C_{pm} becomes the classical potential index of a process, namely $C_{pm} = (USL - LSL)/6\delta$.

Wright risk. Wright (1995) introduced an index which takes into account the asymmetry of the distribution:

$$C_S = \frac{d - |\mu - T|}{3\sqrt{\sigma^2 + (\mu - T)^2 + |\mu_3 / \sigma|}} \quad (16)$$

where T = (LSL + USL)/2, d = (USL – LSL)/2 and $\mu_3 = E(x - \mu)^3$ - the third central moment which describes the departure from symmetry.

Luceño risk. Alberto Luceño proposed the index:

$$C_{pc} = \frac{USL - LSL}{6\sqrt{\pi / 2 * E|X - m|}} \quad (17)$$

where m = (LSL + USL)/2 - this value may be (or may be not) a target value for X.

Crisan risk. D.M. Crişan uses the Kurtosis to correct the potential index, namely:

$$\hat{C} = \frac{USL - LSL}{6s\sqrt{1 + |3 - b_2|}} \quad (18)$$

where $b_2 = \frac{1}{n}\sum (x_i - \bar{x})^4 / s^4$ is the sample coefficient of Kurtosis. We have to notice that the new index takes into account the curve type (platykurtic or leptokurtic as $b_2 < 3$, respectively, $b_2 > 3$, namely with a flatter peak around its mean, which causes thin tails within the distribution, respectively with a higher peak around the mean compared to normal distributions, which leads to thick tails on both sides). Consequently:

$$\hat{C} = \begin{cases} \frac{T_s - T_i}{6s\sqrt{4 - b_2}}, b_2 < 3 (platykurtic) \\ \frac{T_s - T_i}{6s\sqrt{b_2 - 2}}, b_2 > 3 (leptokurtic) \end{cases} \quad (19)$$

In case of normal law, we have $b_2 = 3$ and consequently, the Crisan's index becomes again the classical one $\hat{C}_p$. Barsan-Pipu and Popescu outline an interesting relationship between the Taguchi average loss and the Taguchi potential index, assuming a certain statistical performance level of the NPS process:

$$E[L(X,T)] = K\left|\frac{NPS * \sigma}{C_{pm}}\right|^2 \quad (20)$$

Concrete examples of calculations are given in the cited work – calculating inclusively the costs of such an approach (see pages 107 – 111). ISO document ISO/DIS 3534-2/2004 (ϕ 1.2 "Statistical process management", pages 11 - 30) introduced the so-called reference interval $x_{99.865\%} - x_{0.135\%}$, where $x_{a\%}$ is the a% - fractile of the distribution. This interval is then used to construct "process performance index"

$$P_p = \frac{USL - LSL}{x_{99.865\%} - x_{0.135\%}} \quad (21)$$

which should be used when the process is not in statistical control (for normal distribution, the reference interval is 6σ or 6s). Analogously, one may introduce

$$P_{pk_L} = \frac{x_{50\%} - LSL}{x_{50\%} - x_{0.135\%}} \quad and \quad P_{pk_U} = \frac{x_{50\%} - LSL}{x_{50\%} - x_{0.135\%}} \quad (22)$$

and finally $P_{pk} = min\left\{P_{pk_L}, P_{pk_U}\right\}$

Caulcutt risk. Caulcutt (1995) uses the index (page 155):

$$C_{pu} = \frac{USL - Median}{3s - Median} \quad (23)$$

in order to describe a Skewed distribution.

Vännman risk. It is important to notice that Kerstin Vännman unified some of these indices:

$$C_p(u,v) = \frac{d - u|\mu - M|}{3\sqrt{\sigma^2 + v(\mu - T)^2}} \quad (24)$$

where u, v are real numbers, d = (USL – LSL)/2, M = (LSL + USL)/2, T - target value (which may be M sometimes). Hence, we get easily: $C_p(0, 0) = C_p$ (classical potential index); $C_p(1, 0) = C_{pk}$ (Kane); $C_p(0, 1) = C_{pm}$ (Taguchi); $C_p(1, 1) = C_{pmk}$ (Peern - Kotz - Jonhson).

Operational risk in the banking system

- Some similarities

The banking system is the subject of continuous performance constraints. Failures and operational incidents indicate a risk that can cause the disruption of generate financial flows and, obviously, profit. Between the bank incidents are included: the introduction of counterfeit, cloned cards, informatics attacks, and the use of false identity documents, attacking and robbing of ATMs. A bank operating in a system without incidents may be assimilated to a technical system, smoothly functioning without failures, hence the possibility of make use of the statistical indicators for the behavior of these systems over time.

Isaic-Maniu (2006) and further developed (Isaic-Maniu & Dragan 2010) gives a risk interpretations for the main indicators used in reliability theory; in fact, the distribution function $F_{(t)}$ of a continuous and positive random variable (T) which describes the failure behavior of a given entity may be viewed as a technical risk - that is the complement of the reliability function:

$$Prob\{T < t_0\} = F(t_0) = 1 - R(t_0) = 1 - Prob\{T \geq t_0\} \quad (25)$$

Here $F(t_0)$ is hence the probability that the system operates less than a desired time t_0. If the reliability $R(t_0)$ is low, consequently this technical risk is high. More adequate to define this technical risk seems to be the hazard rate (or failure rate) function which may be called also „the danger of failure":

$$\lambda(t) = \frac{f_{(t)}}{1 - F_{(t)}} = \frac{dF_{(t)}}{R_{(t)}} = -\frac{dR_{(t)}}{R_{(t)}} \quad (26)$$

A high value of means a low level of reliability λ(t) is expressed usually in failures/hour).

- Indicators of operational incidents

Reliability measurement can be achieved by two main groups of indicators: indicators based on descriptive statistical methods (nonparametric indicators) and indicators based on probabilistic models (parametric indicators).

In his monumental paper "Encyclopaedia of Statistical sciences" the well-known statisticians N.L. Johnson and S. Kotz (Johnson & Kotz 1983) state that one of the basic preoccupations of classical statistics was, and still is, the finding of statistic models generation mechanism, in other words finding distribution functions or probability densities.

Accordingly, they enumerate five systems of "frequency curves": Pearson (as differential equation), Gram-Charlier-Edgeworth (as series development), Burr (as differential equation), Johnson (as normality transformation), and Turkey (special transformation).

Also worth mentioning are – in chronological order – the McKay system (1932), based on the use of Bessel functions, the Perks system (1932), which generates a very wide range of logistic distributions, the Darmois (1935) and Koopman (1936) systems regarding exponential distributions, the Gnedenko system (1943) using extreme value distributions, the Toranzos (1952) system, a generalization of the Pearson and Mathai-Saxena systems (1966) based on the generalized hypergeometric function.

In 1982, M. A. Savageau (Savageau 1982) proposes a system of differential equations in which the unknown variables are two densities f_1 and f_2:

$$\begin{cases} \frac{df_1}{dx} = a_1 * f_1^{\alpha_{11}} * f_1^{\alpha_{12}} - b_1 * f_1^{\beta_{11}} * f_2^{\beta_{12}} \\ \frac{df_2}{dx} = a_2 * f_1^{\alpha_{21}} * f_2^{\alpha_{22}} - b_2 * f_1^{\beta_{21}} * f_2^{\beta_{22}} \end{cases} \quad (27)$$

where a_1, a_2, b_1, b_2 are real negative numbers and α_{11}, α_{12}, etc. are random real numbers which provide known densities through various particularizations.

For example, $a_1 = 1$, $a_2 = 1$, $b_1 = 1$, $b_2 = 0$, $\alpha_{11} = 1$, $\alpha_{12} = -1$, $\beta_{11} = 1$, $\beta_{11} = -1$, $\beta_{12} = -1$, $\alpha_{21} = 0$, $\alpha_{22} = 0$, $\beta_{21} = 0$, $\beta_{22} = 0$ leads us to:

$$\frac{df_1}{dx} = \frac{f_1}{f_2} - f_1 * f_2 \quad and \quad \frac{df_2}{dx} = 1,$$

where $f_2 = x$ and therefore

$$\frac{df_1}{dx} = \frac{1}{x} * f_1 - x * f_1 \quad or \quad \frac{df_1}{f_1} = \left(\frac{1}{x} - x\right) * dx,$$

which through integrations leads us to the density

$$f_1(x) = exp\left\{\int\left(\frac{1}{x} - x\right)dx\right\} = exp\left\{ln\,x - \frac{x^2}{2}\right\} = x * exp\left\{-\frac{x^2}{2}\right\}, x \quad 0$$

which is precisely the reduced Rayleigh density.

The majority of the proposed systems are in fact differential equations in which the unknown variable is either F (the distribution function) or f (the probability function).

In general, the density expression depends on the roots of the denominator $c_1 + c_2x + c_3x^2$ and as can be seen in the two examples presented, the c_i coefficients are various moments of the distribution.

A general model. In the followings, we will propose a general differential equation, which has the interesting property that depending on the particular choices of the implicated elements, it provides the distribution function or the probability density, which neither one of the other systems does.

The differential equation is:

$$\frac{d\varphi}{dx} = a(x) * \varphi^{\alpha} + b(x) * \varphi^{\beta}, \quad (28)$$

where φ is a strictly positive real function, for x usually between $(0,+\infty)$, a (x) and b (x) are continuous functions defined on R, and α,β two random numbers.

We will demonstrate that this equation provides distribution functions or density functions, by case. First of all, let us notice that for b(x) = – a(x), $\alpha = 1$ and $\beta = 2$ we have

$$\frac{d\varphi}{dx} = \varphi(1-\varphi) * a(x), \quad (29)$$

which is precisely the Burr model; therefore, this equation can be considered a natural generalization of the Burr model.

The Weibull model. If we chose $a(x) = b(x) = (k/\theta)*x^{k-1}$, θ, $k > 0$, $\alpha = 0$, $\beta = 1$ and determine

$$\frac{d\varphi}{dx} = (1-\varphi)\frac{k}{\theta}x^{k-1} \quad or \quad \frac{d\varphi}{1-\varphi} = \frac{k}{\theta}x^{k-1}$$

meaning that

$$-ln(1-\phi) = \frac{x^k}{\theta}, \quad so\ that \quad 1-\phi = exp\left\{-\frac{x}{\theta}\right\}$$

therefore

$$\varphi(x;\theta,k) = 1 - exp\left\{\frac{-x^k}{\theta}\right\}, \quad x \quad 0, \quad k > 0 \quad (30)$$

which is the well-known Weibull distribution.

Let us look at some historic elements: in 1939, the Swedish engineer Waloddi Weibull, professor at the Royal Institute of Technology in Stockholm (KTH - Kungl Tekniska Högskolan), proposed a statistical model to describe the spread manner of the observed values during experiments regarding the resistance of materials.

He started from the idea that the "fatigue" of the materials cannot be described realistically by the exponential model, since its failure rate does not depend explicitly on time. The model can also be used in the case of banking incidents, due to its general virtues.

In order to avoid this fault of the exponential case, Weibull takes into consideration the following distribution:

$$X : F_x(x;\gamma,\theta,k) = 1 - exp\left\{-\left|\frac{x-\gamma}{\theta}\right|^k\right\}, \quad (31)$$

where $x \geq \gamma > 0$. For the particular case of $\gamma = 0$ and $k = 1$, it contains the exponential model, while for $\gamma = 0$ and $k = 2$, Lord Rayleigh's model (1842-1919).

Let us observe that for Rayleigh's case

$$F_x(x;0,\theta,2) = 1 - exp\left\{-\left|\frac{x}{\theta}\right|^2\right\}, x \quad 0, \theta > 0 \quad (32)$$

the incident intensity is a linear time function $z(x) = (2/\theta) * x$ and that it's obvious that if $x_1 < x_2$ than $z(x_1) < z(x_2)$, and therefore the respective indicator is directly proportional to time. This aspect makes the Rayleigh model be an efficient describer of usual incidents.

Later on, in 1977, Weibull himself commenced a bibliographic research and identified a number of 1019 articles and 36 book titles which dealt with various aspects of the model. Weibull had already expressed his intentions of completing this study through the investigation of other publications besides the ones in English, but his death, in 1979, put an end to these plans.

An important explanation is imposed: Weibull has the merit of highlighting the wide range of applications of the model, but theoretically, it was deduced by the French mathematician Maurice Fréchet back in 1927 (Fréchet 1927). The British statisticians R. A. Fisher and L. H. Tippet deduce it in a study concerning limiting forms of the extreme statistics (Fisher & Tippet 1928). The Russian academician B. V. Gnedenko also studies the model thoroughly through these limiting distributions (Gnedenko 1943). All these papers were purely mathematical, and they didn't have the necessary impact among practicians, while the Weibull Model entered and remained in the post-war scientific literature definitely related to the name of the Swedish engineer. In the francophone literature, also accessible in Romania, the monographic paper of two French engineers benefited from a certain spreading: "Fiabilité et Statistique Previsionelles: Méthodes de Weibull" (Pollard & Rivoire 1971). They were also the ones to introduce the expression "Weibull Method", taken by the Japanese under the term "Waiburumokei" and by the Russians as "Pravila Veibulla - Правила Veibulla", meaning Weibull's Rule.

Thus considering the bi-parametric distribution

$$X : F_x(x;\theta,k) = 1 - exp\left\{-\left|\frac{x}{\theta}\right|^k\right\}, \quad x \quad 0, \quad \theta, k > 0 \quad (33)$$

to which the following density corresponds

$$X : f_x(x;\theta,k) = k\theta^{-k}x^{k-1}exp\left\{-\left|\frac{x}{\theta}\right|^k\right\}, \quad x \quad 0, \quad \theta, k > 0 \quad (34)$$

For this form $E(X) = \theta * \Gamma(1 + 1/k)$ and $-Var(X) = \theta^2 * [\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)]$, $X_{me} = \theta(ln2)^{1/k}$ - the median and $X_{mo} = \theta[(k/1)/k]^{1/k}$ - the mode (k>1).

If the k parameter is relatively large, than we can utilize the approximations:

$E(X) \approx 1 - 057722/\theta + 0{,}98905/\theta^2$ and $Var(X) \approx 1{,}64493/\theta^2$.

We also have $0{,}885*\theta < E(X<\theta)$ and $F_x(\theta) \approx 0{,}63$.

The variation coefficient CV(X) only depends on the k parameter

$$CV(X) = \left[\frac{\Gamma(1+2/k)}{\Gamma^2(1+1/k)} - 1\right]^{1/2}$$

Estimating the model parameters through the ordinary least squares method, or the maximum likelihood method, are the most common procedures.

- Estimating the model parameters based on truncated observations

Because of the fact that the most frequently encountered situation is not based on total observations but on partial ones, whose lengths are interrupted, this being a distinct situation from that of technical systems, observed till exhaustion, we shall consider this case, applicable to banking systems observable for a period of time, followed by assessment.

Thus, if we consider X_0 – the time when observations are interrupted, and $X_{(r)}$ the moment of the rth incident, the likelihood function for this incident variant is described as:

$$L = f_{x_x}\left(X_{\gamma_0+1},...,X_{\gamma}\right)$$

We obtain

$$L=\frac{n!}{(n-r)!r_0}\left|\frac{\beta}{\theta}\right|^{r-r_0} * \prod_{i=r_0+1}^{r}\left|\frac{x_i-\gamma}{\theta}\right|^{\beta-1} * exp\left[-\sum_{i=r_0+1}^{r}\left|\frac{x_i-\gamma}{\theta}\right|^{\beta}-(n-r)\left|\frac{x_i-\gamma}{\theta}\right|^{\beta}\right] \cdot \{1-exp\left[-\left|\frac{t_{r_i+1}-\gamma}{\theta}\right|^{\beta}\right]^{\gamma_0}\} \quad (35)$$

Calculating the derivatives of the logarithm for the previous function, relative to the three parameters, we have

$$\frac{\partial ln L}{\partial \theta} = \frac{(r-r_0)\beta}{\theta} + \beta\frac{\sum_{i=r_0+1}^{r}(x_i-\gamma)^{\beta}}{\theta^{\beta+1}} + \frac{\beta(n-r)(x_r-\gamma)^{\beta}}{\theta^{\beta+1}} - \frac{\beta(x_{r_0+1}-\gamma)^{\beta}}{\theta^{\beta+1}} * \frac{r_0\, exp\left[-(x^{r_0+1}-\gamma)^{\beta}/\theta^{\beta}\right]}{1-exp\left[-(x^{r_0+1}-\gamma)^{\beta}/\theta^{\beta}\right]} \quad (36)$$

for θ, and then for the form parameter

$$\frac{\partial ln L}{\partial \theta} = (r-r_0)\left|\frac{1}{\beta}-ln\,\theta\right| + \sum_{i=r_0+1}^{r} ln(x_i-\gamma) - \sum_{i=r_0+1}^{r}\left|\frac{x_i-\gamma}{\theta}\right| ln\left|\frac{x_i-\gamma}{\theta}\right| - (n-r)\left|\frac{x_r-\gamma}{\theta}\right|^{\beta} * ln\left|\frac{x_r-\gamma}{\theta}\right| + r_0(x_{r_0+1}-\gamma)^{\beta} ln\left|\frac{x_{r_0+1}-\gamma}{\theta}\right| \quad (37)$$

and, finally, for the γ parameter

$$\frac{\partial ln L}{\partial \theta} = (1-\beta)\sum_{i=r_0+1}^{r}(x_i-\gamma)^{-1} + \frac{\beta}{\theta^{\beta}}\sum_{i=r_0+1}^{r}(x_i-\gamma)^{\beta-1} + (n-r)\frac{\beta}{\theta^{\beta}}(x_i-\gamma)^{\beta-1} - \beta_{r_0}(x_{r_0+1}-\gamma)^{\beta-1} * \frac{exp\left[-(x_{r_0+1}-\gamma)^{\beta}/\theta^{\beta}\right]/\theta^{\beta}}{1-exp\left[-(x_{r_0+1}-\gamma)^{\beta}/\theta^{\beta}\right]/\theta^{\beta}} \quad (38)$$

An example for characterizing operational banking incidents

In the period July 1 – September 30 2009, the frequency of banking incidents was registered for a group of three banks with a total of 656 operative units.

The observation time values were transformed in standard work hours (8 hours), while the observations and results registered over the incidents were grouped in six day intervals (48 effective hours of operation), the final results being presented as a series in table 1. The banking incidents observations (false banknotes, cloned cards, false documents, informatics system hacking, the devastation of ATMs etc) concerned three banks in different categories of size:

1. Strong bank with 430 operative units.
2. Middle bank with 180 operative units
3. Small bank with 46 operative units.

The total operational incidents, by hourly intervals, and by category of bank, are presented in table 2; both non-probabilistic indicators and indicators calculated based on a previously validated model shall be determined.

The mean of the overall operational incidents, by type of bank was:

$\bar{k} = 82,85$ - total mean

$\bar{k}_1 = 45,29; \bar{k}_2 = 28,36; \bar{k}_3 = 82,85$

The average number of operational incidents by operational unit is:

$\bar{r} = 1,76$ - general rate

$\bar{r}_1 = 1,47; \bar{r}_2 = 2,21; \bar{r}_3 = 2,63$

Table 1: Banking incident distribution

No.	Intervals of observation	Operational Incidents by bank type			Total operational incidents
	hours	(k_1) 1	(k_2) 2	(k_3) 3	K Total
1	0 - 48	36	19	3	58
2	48 - 96	48	23	7	78
3	96 - 144	38	28	10	76
4	144 - 192	60	30	11	101
5	192 - 240	75	36	10	129
6	240 - 288	42	42	11	95
7	288 - 336	47	35	7	89
8	336 - 384	40	34	8	82
9	384 - 432	46	28	12	86
10	432 - 480	51	34	11	96
11	480 - 528	42	31	11	84
12	528 - 576	39	29	8	76
13	576 - 624	49	12	6	67
14	624 - 672	21	16	6	43
Total		$\Sigma(k_1)$=634	$\Sigma(k_2)$=397	$\Sigma(k_3)$=121	$\Sigma(k)$=1160

Source: author

The main conclusion that can be drawn from interpreting the results concerns the high reliability of large banks, as the number of operational incidents is lower.

Establishing operational reliability indicators requires achieving a probabilistic model regarding the occurrence of incidents.

The relatively high frequency of the incidents excludes the Poisson model (for the incidents frequency) or the exponential model (for the time period when events occur) and indicates as viable the Weibull model, which includes as particular cases other models, through its generalizing qualities.

For the data in table 2 regarding the operational incidents distribution, the calculations lead to a mean of the operational time $\bar{t} = 323,35$ hours (in operational hours), with a standard deviation of $\hat{s} = 289,3$ hours.

Applying the ordinary least squares method for the Weibull model parameters, in the tri-parametric variant, we have:

$\hat{\theta}$ = 357,65 hours

γ = 6,58 hours

ß = 2,1

The distribution function for the operational reliability model, for t = 240 hours, it is considered that the effective work time in a month is:

$$\hat{F}(t) = 1 - exp\left[-\left|\frac{t-6,58}{337,65}\right|^{2,1}\right] = 0,335$$

The reliability function for an operational monthly interval:

$$\hat{R}(t) = 1 - exp\left[-\left|\frac{t-6,58}{337,65}\right|^{2,1}\right] = 0,6648$$

The intensity of the banking operational incidents:

$$\hat{r}(t) = \frac{2,1(240-6,58)^{2,1-1,0}}{357,65^{2,1}} = 0,00367$$

The final conclusion concerns the low reliability of the system of the three banks under observation. With 66.5% chance of surpassing the 240 hour period of functioning without operational incidents, the system of the three banks proves to be extremely vulnerable.

The calculations were determined on the system of the three banks, and for smaller banks, reliability proves to be about 0.6 times lower as against to larger banks.

This fact demands increased preventive measures for this group of banks, parallel to a general preoccupation of reducing operational incidents.

Conclusion

The management process is strongly linked to the micro and macroeconomic environment, full of uncertainties and the decisions are affected by risks. Mainly, these are occurring especially in decision-making, and as they affect only one of the parts, they generate different risks, statistically measured by β – type II risk.

These risks arise and are of importance in the company relations, with respect to external environment, decision-making on contracts deliveries, purchases, economic exchanges of goods and services etc. As for the management internal process, the control of processes (i.e.: the manufacturing processes in a factory, the bureaucratic processes in a bank or in administrative institutions, the flow processes in a hospital etc.) are subjected to a certain risk, probabilistically measured, to get out of normal limits. This issue is measured by its capacity and exceeding the capability limits is measured by different risks as Taguchi, Waright, only to cite the most known ones. According to the process state, a whole literature was developed, that provides solutions to the diversity of practical situations which are of real help in the operational management of processes.

References

Borgonovo, E., “A Methodology for Determining Interactions in Probabilistic Safety Assessment Models by Varying One Parameter at a Time”, Risk Analysis, 30(3):385-99, 2010.

Caulcutt, R. Achieving Quality Improvement. A Practical Guide, Chepman and Hall, London, 1995.

Chan, L. K., Cheng, S.W. and Spiring, F. A. “A New Measure of Process Capability: Cpm”, Journal of Quality Technology, 20 (3): 162-175, 1988.

Christoph H. L., Michael E. S. and Elaine M. B., „Diagnosing Unforeseeable Uncertainty in a New Venture”, Journal of Product Innovation Management, 25(1): 28–46, 2008.

Crowder, M., “Characterizations of Competing Risks in Terms of Independent-Risks Proxy Models”, Scandinavian Journal of Statistics, 27(1): 57–64, 2000.

Gogoneata, B., Economia riscului si incertitudinii, Editura Economica, Bucuresti, 2004.

Hopkin, P., Fundamentals of Risk Management, 2nd ed., Kogan-Page, 2012.

Isaic-Maniu, A., Dragan, I. M., “The risk of operational incidents in banking institutions”, Reliability &Risk Analysis: Theory & Applications, 1(16) vol.1: 121-127, USA, 2010.

Isaic-Maniu, I. Caracterizarea statistica a riscului, ASE Publishing, 2006.

Kackar, R. N. “Off-line Quality Control, Parameter Design and the Taguchi Method”, Journal of Quality Technology, 17(4): 176-188, 1985.

Lindley, D. “The Philosophy of Statistics”, Journal of the Royal Statistical Society, Series D 49 (3): 293-337, 2000.

Lovric, M. (edit.) International Encyclopedia of Statistical Science, Springer, 2011.

Stoica, M., Management Bancar, Editura Economica, Bucuresti, 1999.

Uyemura, D. G., Deventa, D. R., Financial Risk Management in Banking, Wiley, London, 1993.

Vose, D. Risk Analysis: A Quantitative Guide, John Wiley & Sons, London, 2008.

Voda, V. “Some Comments on Statistical Risks”, Reliability &Risk Analysis: Theory & Applications, 1(12) vol.2: 121-127, USA, 2009.

Wright, A. P. “A Process Capability Index Sensitive to Skewness”, Journal of Statistical Computation and Simulation, 52: 195-203, 1995.