FROM MODIGLIANI-MILLER TO GENERAL THEORY OF CAPITAL COST AND CAPITAL STRUCTURE OF THE COMPANY

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ABSTRACT

One of the serious limitations of the Modigliani–Miller theory is the suggestion about perpetuity of the companies. We lift up this limitation and show, that the accounting of the finite lifetime of the company leads to change of the equity cost, k_e , as well as of the weighted average cost of capital, WACC, in the presence of corporative taxes. We give a rigorous proof of the Brusov–Filatova theorem, that in the absence of corporative taxes cost of company equity, k_e , as well as its weighted average cost, WACC, do not depend on the lifetime or age of the company. We show that perpetuity Modigliani-Miller theory underestimates the equity cost k_e , as well as the weighted average cost of capital, WACC, and thus underestimates the financial risks, which could become one of the implicit reasons for the financial crisis

JEL CLASSIFICATION & KEYWORDS

■ G3 ■ G11 ■ G32 ■ G33 ■ Perpetuity Companies ■ Companies with Finite Lifetime ■ Companies of Arbitrary Age ■ Weighted Average Cost ■ Equity Cost ■ Modigliani-Miller Theory ■ Brusov-Filatova Theorem

INTRODUCTION

Until now, the basic theory of the cost of capital of companies was the theory of Nobel Prize winners Modigliani and Miller. One of the serious limitations of the Modigliani–Miller theory is the suggestion about perpetuity of the companies. We lift up this limitation and show, that the accounting of the finite lifetime of the company leads to change of the equity cost, k_e , as well as of the weighted average cost of capital WACC in the presence of corporative taxes. The effect of leverage on the cost of equity capital of the company, k_e , with an arbitrary lifetime, and its weighted average cost of WACC is investigated. We give a rigorous proof of the Brusov–Filatova theorem, that in the absence of corporative taxes cost of company equity, k_e , as well as its weighted average cost, WACC, do not depend on the lifetime of the company.

I. Companies with arbitrary lifetime

Let us consider the situation with finite lifetime companies. First of all we will find the value of tax shields, TS , of the company for n years

$$TS = k_d DT \sum_{t=1}^{n} (1 + k_d)^{-t} = DT \left[1 - (1 + k_d)^{-n} \right]$$
(1)

(We used the \bar{l} formula for the sum of n terms of a geometric progression).

Here, D is the value of debt capital; k_d – the cost of debt capital, T – income tax rate.

Next, we use the Modigliani – Miller theorem [3,4]:

The value of financially dependent company is equal to the value of the company of the same risk group used no leverage, increased by the value of tax shield arising from financial leverage and equal to the product of rate of corporate income tax T and the value of debt D.

$$V = V_0 + DT \tag{2}$$

This theorem was formulated by Modigliani and Miller for perpetuitive companies, but we apply it for a company with a finite lifetime.

$$V = V_0 + TS = V_0 + k_d DT \sum_{t=1}^{\infty} (1 + k_d)^{-t} =$$

$$= V_0 + w_d VT \left[1 - \left(1 + k_d \right)^{-n} \right]$$
(4)

$$V(1 - w_d V T[1 - (1 + k_d)^{-n}]) = V_0$$
 (5)

There is a common use of the following two formulas for the cost of the financially independent and financially dependent companies [3,4]

$$V_0 = CF/k_0 \text{ and } V = CF/WACC$$
 (6)

However, these almost always used formulas were derived for perpetuitive company and in case of a company with a finite lifetime they must be modified in the same manner as the value of tax shields [1,2]

$$V_0 = CF \left[1 - \left(1 + k_0 \right)^{-n} \right] / k_0;$$

$$V = CF \left[1 - \left(1 + WACC \right)^{-n} \right] / WACC$$
(7)

From formula (5) we get Brusov–Filatova equation for WACC[1,2]

$$\frac{\left[1 - \left(1 + WACC\right)^{-n}\right]}{WACC} = \frac{\left[1 - \left(1 + k_0\right)^{-n}\right]}{k_0 \left[1 - \omega_d T \left(1 - \left(1 + k_d\right)^{-n}\right)\right]}$$
(8)

Here, S – the value of own (equity) capital of the company,

$$\begin{aligned} w_d &= \frac{D}{D+S} & - \text{ the share of debt capital;} \\ k_e, w_e &= \frac{S}{D+S} - \text{ the cost and the share of the equity} \end{aligned}$$

At n=1 we get Myers (Myers, 1974) formula for one-year company

financial leverage

$$WACC = k_0 - \frac{(1+k_0)k_d}{1+k_d} w_d T$$
 (9)

L = D/S

For n = 2 one has

$$\frac{\left[1-\left(1+WACC\right)^{-2}\right]}{WACC} = \frac{\left[1-\left(1+k_{0}\right)^{-2}\right]}{k_{0}\left[1-\omega_{d}T\left(1-\left(1+k_{d}\right)^{-2}\right)\right]} \tag{21}$$
 This equation can be solved for WACC analytically:

$$WACC = \frac{1 - 2\alpha \pm \sqrt{4\alpha + 1}}{2\alpha} \tag{22}$$

where

$$\alpha = \frac{2 + k_0}{(1 + k_0)^2 \left[1 - \omega_d T \frac{2k_d + k_d^2}{(1 + k_d)^2} \right]}$$
(23)

For n = 3 and n = 4 equation for the WACC becomes more complicate, but it still can be solved analytically, while for n > 4 it can be solved only numerically.

We would like to make an important methodological notice: taking into account the finite life-time of the company, all formulas, without exception, should be received with use formulas (18) instead of their perpetuity limits (17).

Below, we will describe the algorithm for the numerical solution of the equation (19).

II. Algorithm for finding of WACC in case of arbitrary life-time of the project

Let us return back to n-year project (n-year company). We have the following equation for WACC in n-year case

$$\frac{\left[1 - \left(1 + WACC\right)^{-n}\right]}{WACC} - A(n) = 0,$$
(24)

where

$$A(n) = \frac{\left[1 - (1 + k_0)^{-n}\right]}{k_0 \left[1 - \omega_d T \left(1 - (1 + k_d)^{-n}\right)\right]}$$
(25)

The algorithm of the solving of the equation (24) should be as following:

- 1. Putting the values of parameters k_0, ω_d, T and given n, we calculate A(n):
- We determine two WACC values, for which the left part of the equation (24) has opposite signs. It is obviously that as these two values we can use $WACC_1$ and $WACC_{\infty}$, because $WACC_{1} > WACC_{n} > WACC_{\infty}$ for finite $n \ge 2$.
- Using, for example, the bisection method, we can solve the equation (24) numerically.

III. Modigliani-Miller (perpetuity company), Myers (oneyear company) and Brusov-Filatova (two-, three-,fiveand ten-year companies) results

Myers (Myers, 1974) has compared his result for one-year project [formula (11)] with Modigliani and Miller's result for perpetuity limits (8). He has used the following values of

 $k_0 = 8\% \div 24\%$; $k_d = 7\%$; T = 50%; $w_d = 0\% \div 60\%$ and estimated the difference in the WACC values following from the formulas (11) and (8). We did make the similar calculations for two-, three-, five- and ten-year project for the same set of parameters and we have gotten the following results, shown in Table.I (second line (bulk)), Table.II

(second line (bulk)) and Table. III and corresponding Figures

Table.I. WACC dependence on debt share \mathcal{W}_d for different values of equity cost k_0 for companies with different lifetime n.

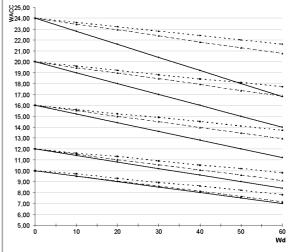
		1					
$k_{\scriptscriptstyle 0}$	n	$w_d = 10\%$	20%	30%	40%	50%	
$k_0 = 8\%$	n=1	7.6%	7.3	6.9	6.6	6.2	5.9
"	n=2	7.52	7.08	6.6	6.17	5.67	5.21
	$n=\infty$	7.6	7.2	6.8	6.4	6.0	5.6
$k_0 = 10\%$	n=1	9.7	9.3	8.9	8.6	8.2	7.8
ů	n=2	9.51	9.05	8.59	8.13	7.64	7.16
	$n=\infty$	9.5	9.0	8.5	8.0	7.5	7.0
$k_0 = 12\%$	n=1	11.6	11.3	10.9	10.5	10.2	9.8
Ů	n=2	11.51	11.02	10.54	10.07	9.6	9.09
	n=3	11.46	10.93	10.39	9.85	9.31	8.77
	n=5	11.42	10.83	10.25	9.66	9.06	8.46
	n=10	11.3964	10.7863	10.1695	9.5455	8.914	8.2745
	$n=\infty$	11.4	10.8	10.2	9.6	9.0	8.4
$k_0 = 16\%$	n=1	15.62	15.2	14.9	14.5	14.1	13.7
ľ	n=2	15.52	14.99	14.5	13.98	13.47	12.96
	n=3	15.44	14.88	14.31	13.75	13.18	12.61
	n=5	15.38	14.76	14.14	13.51	12.88	12.24
	n=10	15.34	14.67	13.99	13.31	12.62	11.92
	$n=\infty$	15.2	14.4	13.6	12.8	12.0	11.2
$k_0 = 20\%$	n=1	19.6	19.2	18.8	18.4	18.1	17.7
_	n=2	19.45	18.97	18.45	17.93	17.37	16.86
	n=3	19.41	18.82	18.23	17.64	17.05	16.45
	n=5	19.35	18.69	18.03	17.36	16.70	16.03
	n=10	19.27	18.54	17.80	17.05	16.30	15.54
	$n=\infty$	19.0	18.0	17.0	16.0	15.0	14.0
$k_0 = 24\%$		23.6	23.2	22.8	22.4	22.0	21.6
	n=2	23.46	22.94	22.37	21.80	21.30	20.75
	n=3	23.39	22.77	22.15	21.54	20.91	20.29
	n=5	23.31	22.61	21.91	21.21	20.51	19.80
	n=10	23.21	22.40	21.60	20.78	19.96	19.13
	$n=\infty$	22.8	21.6	20.4	19.2	18.0	16.8

Source: Authors

Note, that data for equity cost $k_0 = 8\%$ turn out to be a little bit uncertain: this could be relate to the fact that this value of equity cost is quite close to value of interest rate of the debt $k_d = 7\%$. For all other values of equity cost the results are reproducible and very informative and are discussed below

For a graphic illustration of the results, we use data for $n = 1, 2, \infty$, that adequately reflect the results we obtained.

Fig. 1. The dependence of the WACC on debt share W_d for companies with different lifetimes for different cost of equity, k_0 (from Table 1).



Source: Authors

Table II. Dependence of the differences $\Delta_1 = WACC_1 - WACC_{\infty}$ (first line), $\Delta_2 = WACC_1 - WACC_2$ (second line (bulk)) and their ratio $r = \Delta_1/\Delta_2$ (third line) on debt

share w_d for different values of equity cost k_0 .

		000/	0001	100/	5004	000/
	$w_d = 10\%$	20%	30%	40%	50%	60%
$k_0 = 10\%$	0.20	0.30	0.4	0.60	0.7	0.8
~	0.19	0.25	0.31	0.47	0.56	0.64
	1.05	1.2	1.29	1.28	1.25	1.25
$k_0 = 12\%$	0.2	0.5	0.7	0.9	1.2	1.4
	0.09	0.28	0.36	0.43	0.6	0.71
	2.22	1.76	1.94	2.09	2	1.97
$k_0 = 16\%$	0.4	0.8	1.3	1.7	2.1	2.5
_	0.08	0.21	0.4	0.52	0.63	0.74
	5.0	3.81	3.25	3.27	3.33	3.38
$k_0 = 20\%$	0.6	1.2	1.8	2.4	3.1	3.7
	0.15	0.23	0.35	0.47	0.73	0.84
	4.0	5.22	5.14	5.11	4.25	4.4
$k_0 = 24\%$	0.8	1.6	2.4	3.2	4.0	4.8
	0.14	0.26	0.43	0.6	0.7	0.8500
	5.7	6.15	5.58	5.33	5.71	5.6500

Source: Authors

Fig.2. Dependence of the ratio $r = \Delta_1/\Delta_2$ of differences $\Delta_1 = WACC_1 - WACC_{\infty}$, $\Delta_2 = WACC_{-1} - WACC_{-2}$ on debt share W_d for different values of equity cost k_0 (from Table II).

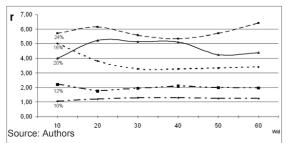
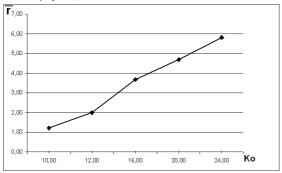


Table III. Dependence of the average ratios $\bar{r} = \langle \Delta_1/\Delta_2 \rangle$ on equity cost k_0 .

k_0	10%	12%	16%	20%	24%
$\bar{r} = <\Delta_1/\Delta_2>$	1.22	2.00	3.67	4.69	5.69

Source: Authors

Fig.3. Dependence of the average values of ratio $\bar{r} = <\Delta_1/\Delta_2>$ on the equity cost, k_0 .



Source: Authors

IV. Discussion of results

1. From Table 1 and Fig.2 it is obviously that WACC has a maximum for one year project and decreases with the life-time of the project, reaching the minimum in the Modigliani-Miller perpetuity case. Dependence of all WACC values on debt share w_d turn out to be linear at any equity cost k_0 for all considered life–time values of the project. It is natural for one-year project, because it is described be Myers linear formula (11) as well as in the Modigliani-Miller perpetuity case, describing by the formula (8), which is linear too, but it is surprised for two-year project, where formula for WACC (22) is obviously nonlinear.

The negative slope in WACC increases with the equity cost

- 2. As it follows from the Table 2 and Fig.3 the dependence of the average ratios $\bar{r} = \langle \Delta_1/\Delta_2 \rangle$ on debt share w_d is guite weak and can be considered as almost constant. The value of this constant increases practically linear with the equity cost k_0 from 1.22 at $k_0 = 10 \%$ up to 5.69 at $k_0 = 24\%$ (see Fig.4).
- 3. The relative difference between one-year and two-year projects increases when the equity cost k_0 decreases. This means that the error in one-year project WACC for two-year project increases when the equity cost k_0 decreases as well. At the same time the relative difference between two-year project and perpetuity MM project increases with the equity $cost k_0$.

V. Brusov-Filatova theorem (case of absence of corporate taxes)

Modigliani-Miller theory in case of absence of corporate taxes gives the following results for dependence of WACC and equity cost k_e on leverage

1.
$$V_0 = V_L$$
; $CF/k_0 = CF/WACC$ and thus $WACC = k_0$ (18)

2.
$$WACC = w_a \cdot k_a + w_d \cdot k_d$$
: and thus

$$k_{e} = \frac{WACC - w_{d} \cdot k_{d}}{w_{e}} = \frac{k_{0} - \frac{L}{1 + L} k_{d}}{\frac{1}{1 + L}} = k_{0} + L(k_{0} - k_{d})$$
(19)

For the finite lifetime companies Modigliani-Miller theorem about equality of value of financially independent and financially dependent companies $(V_0 = V_L)$ has the following view [1,2]

$$V_{0} = V_{L}; CF \cdot \frac{\left[1 - (1 + k_{0})^{-n}\right]}{k_{0}} = CF \cdot \frac{\left[1 - (1 + WACC)^{-n}\right]}{WACC}$$
(20)

Using this relation, we prove an important Brusov-Filatova theorem:

Under absence of corporate taxes the equity cost of the company, k_e , as well as its weighted average cost of capital, WACC, do not depend on the lifetime of the company and are equal respectively to

$$k_e = k_0 + L(k_0 - k_d); WACC = k_0$$
 (21)

Let us consider first the one- and two-year companies a) for one- year company one has from (20)

$$\frac{\left[1 - \left(1 + k_0\right)^{-1}\right]}{k_0} = \frac{\left[1 - \left(1 + WACC\right)^{-1}\right]}{WACC}$$
 (22)

and thus
$$\frac{1}{1+k_0} = \frac{1}{1+WACC}$$
 (23)

Hence
$$WACC = k_0$$
 (24)

Formula for equity cost $k_e = k_0 + L(k_0 - k_d)$ now is obtained from equation (19) because $WACC = k_0$.

b) for two-years company one has from (20)

$$\frac{\left[1 - \left(1 + k_0\right)^{-2}\right]}{k_0} = \frac{\left[1 - \left(1 + WACC\right)^{-2}\right]}{WACC},$$

and thus

$$\frac{2+k_0}{(1+k_0)^2} = \frac{2+WACC}{(1+WACC)^2}$$
 (25)

Denoting $\alpha = \frac{2 + k_0}{(1 + k_0)^2}$, we get the following quadratic

equation for WACC:

$$\alpha \cdot WACC^{2} + (2\alpha - 1) \cdot WACC + (\alpha - 2) = 0$$
 (26)

It has two solutions

$$WACC_{1,2} = \frac{1 - 2\alpha \pm \sqrt{4\alpha + 1}}{2\alpha}.$$
 (27)

Substituting $\alpha = \frac{2 + k_0}{(1 + k_0)^2}$, we get

$$WACC_{1,2} = \frac{(k_0^2 - 3) \pm (k_0 + 3)(1 + k_0)}{2(2 + k_0)}.$$
 (28)

$$WACC_1 = k_0$$
; $WACC_2 = -\frac{2k_0 + 3}{k_0 + 2} < 0$. (29)

The second root is negative, but the weighted average cost of capital can only be positive, so only one value remains

$$WACC_1 = k_0$$
.

c) For company with arbitrary lifetime n Brusov-Filatova formula (20) gives

$$\frac{\left[1 - \left(1 + k_0\right)^{-n}\right]}{k_0} = \frac{\left[1 - \left(1 + WACC\right)^{-n}\right]}{WACC}$$
(30)

For a fixed k_0 (30) is an equation of n-degree relative to WACC. It has *n* roots (in general complex). One of the roots, as shown by direct substitution, is always $WACC = k_0$. Investigation of the remaining roots is difficult and not part of our problem.

Formula for equity cost $k_e = k_0 + L(k_0 - k_d)$ now is obtained from equation (19) because $WACC = k_0$.

Thus we have proved the Brusov-Filatova theorem.

VI. Case of the presence of corporate taxes

Modigliani-Miller theory in case of presence of corporate taxes gives the following results for dependence of WACC and equity cost k_e on leverage

1) WACC

$$V_L = V_0 + Dt ; D = w_d V_L;$$
 (31)

$$CF/WACC = CF/k_0 + Dt = CF/k_0 + w_d t CF/WACC$$
 (32)

$$\left| \frac{1 - w_d t}{WACC} = \frac{1}{k_0}, \right| \tag{33}$$

$$WACC = k_0 (1 - w_d t) = k_0 (1 - \frac{L}{1 + L} t)$$
 (34)

Thus, WACC decreases with leverage from k_0 (in the absence of debt financing (*L*=0)) up to $^{k_0(1-t)}$ (at $^{L}=\infty$). 2) The equity cost k_e

$$WACC = k_0(1 - w_d t) = w_e \cdot k_e + w_d \cdot k_d(1 - t);$$

and thus

$$k_{e} = \frac{WACC - w_{d} \cdot k_{d} \cdot (1-t)}{w_{e}} = \frac{k_{0}(1-w_{d}t) - \frac{L}{1+L}k_{d}(1-t)}{\frac{1}{1+L}} = k_{0} + L(k_{0} - k_{d})(1-t).$$
(35)

Let us consider how the weighted average cost of capital, WACC, and the cost of equity capital, k_e , will be changed under taking into account the finite lifetime of the company.

a) One-year company

From (20) one has

$$\frac{\left[1 - \left(1 + WACC\right)^{-n}\right]}{WACC} = \frac{\left[1 - \left(1 + k_0\right)^{-n}\right]}{k_0 \left[1 - w_d t \left(1 - \left(1 + k_d\right)^{-n}\right)\right]}$$
(36)

For one-year company we get

$$\frac{\left[1 - (1 + WACC)^{-1}\right]}{WACC} = \frac{\left[1 - (1 + k_0)^{-1}\right]}{k_0 \left[1 - w_d t \left(1 - (1 + k_d)^{-1}\right)\right]}$$
(37)

From (37) we obtain the well-known Myers formula (9), which is the particular case of Brusov-Filatova formula (20).

$$WACC = k_0 - \frac{1 + k_0}{1 + k_d} k_d w_d t$$

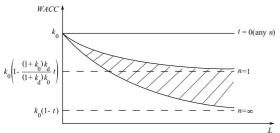
Thus

$$WACC = k_0 \left(1 - \frac{\left(1 + k_0 \right) \cdot k_d}{\left(1 + k_d \right) \cdot k_0} \cdot \frac{L}{1 + L} t \right)$$
(38)

Thus, WACC decreases with leverage from k_0 (in the absence of debt financing (L=0)) up to

$$k_0 \left(1 - \frac{\left(1 + k_0 \right) \cdot k_d}{\left(1 + k_d \right) \cdot k_0} t \right) \text{ (at } L = \infty \text{)}.$$

Fig.4. The dependence of the WACC on leverage in the absence of corporate taxes (the horizontal line (t=0)), as well as in the presence of corporate taxes (for one–year (n=1) and perpetuitive companies $(n=\infty)$). Curves for the WACC of companies with an intermediate lifetime $(1 < n < \infty)$ lie within the shaded region.



Source: Authors

Equating the right part to general expression for WACC $WACC = w_e \cdot k_e + w_d \cdot k_d \left(1 - t \right) \tag{39}$ one gets

$$k_0 - \frac{1 + k_0}{1 + k_d} k_d w_d t = w_e \cdot k_e + w_d \cdot k_d (1 - t)$$
 (40)

Thus

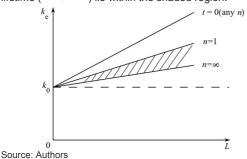
$$\begin{split} k_e &= \frac{1}{w_e} \Bigg[k_0 - \frac{1 + k_0}{1 + k_d} \, k_d w_d t - k_d w_d \big(1 - t \big) \Bigg] = \\ &= \big(1 + L \big) k_0 - L \frac{k_d}{1 + k_d} \Big[\big(1 + k_0 \big) t + \big(1 + k_d \big) \big(1 - t \big) \Big] = \\ &= k_0 + L \big(k_0 - k_d \bigg) \Bigg(1 - \frac{k_d}{1 + k_d} t \Bigg). \end{split}$$

$$k_e = k_0 + L \big(k_0 - k_d \bigg) \Bigg(1 - \frac{k_d}{1 + k_d} t \Bigg). \tag{41}$$
 So, we see, that in case of one–year company the perpetuity

So, we see, that in case of one–year company the perpetuity limit $k_e=k_0+L\left(k_0-k_d\right)\!\left(1-t\right)$ is replaced by (41).

Difference is due to different values of the tax shield for a one—year company and perpetuitive one.

Fig.5. Dependence of the equity cost, k_e , on leverage in the absence of corporate taxes (the upper line (t=0)), as well as in the presence of corporate taxes (for one–year (n=1) and perpetuitive companies ($n=\infty$)). Dependences of the cost of equity capital of companies, k_e , with an intermediate lifetime $(1 < n < \infty$) lie within the shaded region.



Let us investigate the question of the tax shields value for companies with different life–time in more detail.

VII. Tax shield

General expression for the tax shield has the form (Brusov–Filatova)

$$TS = \sum_{i=1}^{n} \frac{k_d Dt}{(1+k_d)^i} = \frac{k_d Dt \left[1-(1+k_d)^{-n}\right]}{(1+k_d)(1-(1+k_d)^{-1})} = Dt \left[1-(1+k_d)^{-n}\right]. (42)$$

1) In perpetuitive limit $(n \to \infty)$ tax shield is equal to $TS_\infty = Dt$, which leads to the so–called effect of the tax shield associated with the appearance of a factor (1-t) in the equity cost $k_\varepsilon = k_0 + L(k_0 - k_d)(1-t)$.

2) For the one–year company tax shield value is equal to

$$TS_1 = Dt \left(1 - \left(1 + k_d \right)^{-1} \right) = Dtk_d / (1 + k_d)$$
 (43)

This leads to appearence of a factor $\left(1-\frac{k_d}{1+k_d}t\right)$ in the equity

cost
$$k_e = k_0 + L(k_0 - k_d) \left(1 - \frac{k_d}{1 + k_d} t \right)$$
 (41)

3) tax shield for a two-year company is equal to

$$TS_2 = Dt(1 - (1 + k_d)^{-2}) = Dtk_d(2 + k_d)/(1 + k_d)^2$$
 (44)

and if the analogy with one–year company will keep, then factor (1-t) in the Modigliani–Miller theory would be replaced by the factor

$$\left(1 - \frac{k_d(2 + k_d)}{(1 + k_d)^2} t\right)$$
(45)

However, due to a nonlinear relation between WACC and $k_{\rm 0}$ and $k_{\rm d}$ in Brusov–Filatova formula (15) for two–year company (and companies with longer life–time), such a simple analogy is no longer observed, and the calculations become more complex.

CONCLUSION

In the paper an important step towards a general theory of capital cost and capital structure of the company has been done. For this perpetuity theory of Nobel Prize winners Modigliani and Miller, which is still the basic theory of capital cost and capital structure of companies, extended to the case of companies with an arbitrary lifetime, as well as for companies of arbitrary age.

We show that taking into account the finite lifetime of the company in the presence of corporate taxes leads to a change in the equity cost of the company, $k_{\rm e}$, as well as in its weighted average cost, WACC.

Thus, we have removed one of the most serious limitations of the theory of Modigliani–Miller connected with the assumption of perpetuity of the companies. The effect of leverage on the cost of equity capital of the company with

an arbitrary lifetime, k_e , and its weighted average cost, WACC, is investigated. We give a rigorous proof of an important Brusov–Filatova theorem, that in the absence of corporate tax equity cost of companies, k_e , as well as its weighted average cost, \it{WACC} , do not depend on the lifetime of the company.

Incorrect assessment of key financial parameters of companies within perpetuity Modigliani-Miller theory leads to an underestimation of the financial risks, inability, or serious difficulties in making management decisions, which could become one of the implicit reasons for the financial crisis

From the other side use of modern theory, suggested by authors, gives a more adequate assessment. This may help avoid future financial crises, as companies will realistically assess their financial situation.

In conclusion we give a Table IV, which shows the difference between results of Modigliani-Miller theory (already particular) and ones of general theory. It is obvious, that difference is so significant that one should always use the results of new theory.

Table IV. Difference between results of Modigliani-Miller theory and ones of general theory.

	Modigliani-Miller theory results	Brusov-Filatova
parameters		theory results
Capitalization	$V_{r} = CF/k_{r}$	CF[1
of an	, , ,,,,,	$V_0 = \frac{CF}{k_*} [1 - (1 + k_0)^{-n}]$
financially		k, ' ' '
independent		0
company		
Capitalization	V = CF/WACC	CF [\sim 1.
of an		$V = \frac{CF}{WACC} \left[1 - (1 + WACC)^{-n} \right]$
		WACC
		, , , , , , , , , , , , , , , , , , , ,
Tax shield	(TS) = DT	$(TS) = DT \left[1 - (1 + k_A)^{-n}\right]$
	\ /	
	$V = V_0 + DT$	$V = V_0 + DT \left[1 - (1 + k_d)^{-n} \right]$
	,	
	$WACC = k_0(1 - w_s t)$	$1 - (1 + WACC)^{-n}$
	0\ 4 /	
		WACC
WACC		[/ - \ "]
		$[1-(1+k_0)^{-n}]$
		= 7 1
		$=\frac{\left[1-(1+k_0)^{-n}\right]}{k_0\left[1-\omega_dT\left(1-(1+k_d)^{-n}\right)\right]}$
Cost of equity,	$k_o = k_0 + L(k_0 - k_d)(1-t)$	$k_e = (1 + L)WACC -$
k_{a}	" " " "	
		$-Lk_d(1-t)[1-(1+k_d)^{-n}]$
	Capitalization of an financially independent company Capitalization of an financially dependent company Tax shield Modigliani-Miller theorem Weighted average cost of capital, WACC	parameters $V_0 = CF/k_0$ framework from the first parameter of an financially company $Capitalization of an financially dependent company Tax \text{ sheld} \qquad V = CF/WACC for a financially dependent company Tax \text{ sheld} \qquad (TS) = DT Modiglian-Miller theorem V = V_0 + DT Weighted average cost of capital, WACC WaCC = k_0(1 - w_d t) Cost of equily, k_x = k_0 + L(k_0 - k_s)(1 - t)$

Source: Authors

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