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INFLUENCE OF DEBT FINANCING ON THE EFFECTIVENESS OF THE INVESTMENT PROJECT WITHIN THE MODIGLIANI–MILLER THEORY

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ABSTRACT

We study the problem of the influence of debt financing on the effectiveness of the investment project within the theory of Modigliani-Miller and obtain quantitative results for the first time. The effectiveness of the investment project is considered from two perspectives: the owners of equity and debt and equity holders only. It was shown, that NPV practically always decreases with leverage in case of a constant value of equity, and the maximum leverage level, at which the project is still effective (NPV>0), was found. In case of a constant value of the total invested capital it is possible an increase of NPV with leverage as well as its decrease, depending on the relation between the parameters of the project.

JEL CLASSIFICATION & KEYWORDS

■ G31 ■ E22 ■ G00 ■ C02 ■ Effectiveness of the Investment Project ■ Modigliani–Miller Theory ■ Debt Financing

INTRODUCTION

Under the capital structure in the context of investments one understands the relation between the magnitude of debt and equity funds invested in the project, and because, as a rule, projects are invested by own and borrowed funds, the problem of the effect of the debt financing on the effectiveness of the investment project is extremely urgent. Hope to determine the optimal capital structure, in which one or more parameters of effectiveness (NPV, IRR, etc.) are maximized, leads investigators to engage in this issue during long time.

In this paper for the first time during many years of research of the problem of the influence of debt financing on the effectiveness of the investment project tangible results have been obtained within the theory of Modigliani–Miller.

1. General remarks

There is an alternative: to include or not include credit flows in the evaluation of the project by NPV. The recommendation does not take into account the flow of credit is valid in the absence of interaction between investment and credit decisions and has the following arguments (Kuznetsova and Livshits (1995)):

1. get a credit can be considered as a separate event because of its independence from the project;
2. cash flows should be discounted at a rate commensurate with their degree of risk (i.e. the extent of uncertainty), so credit flows are discounted at the interest rate k_d (lenders evaluate their loan, taking into account the degree of risk and have the same cash flow with the opposite sign and with the same degree of uncertainty).
3. credit flows in the absence of interaction with the project are a credit, interest and repayment of a principal. Their discounted value in NPV at discounting rate k_d is equal to zero.

In a real financial market credit can not be considered as a separate event and NPV of a credit is not zero, and therefore, when evaluating the effectiveness of project credit flows must be considered. There are several methods of taking them into account when assessing the NPV, and they are all associated with the choice of one or more effective discount rates. A similar choice is connected with the calculation of IRR, which may have a few modifications.

Methods of the NPV finding can be grouped in two directions (Kuznetsova and Livshits (1995)).

1. Determination by the investor an accurate discounting rate, k_e^*, addressing the effects of debt financing, which would allow him not to divide flows to the financial and operating plus investment. Then

$$NPV_L = \sum_{i=0}^{n} \frac{P_i + F_i}{(1 + k_e^*)^i} \quad (1)$$

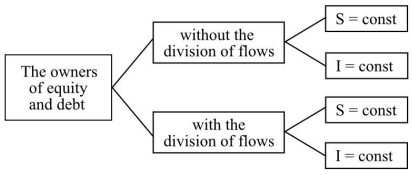
2. Separate financial flows from operating and investment ones and discount each component at its own discount rate: operating, investment flows are discounted at a rate k_e and credit flows – at the rate k_d. Then the decision is getting by the value

$$NPV_L = \sum_{i=0}^{n} \frac{P_i}{(1 + k_e)^i} + \sum_{i=0}^{n} \frac{F_i}{(1 + k_d)^i} \quad (2)$$

Note, that in the first method of finding the NPV, it seems to be reasonably to use as a discounting rate the weighted average cost of capital, WACC.

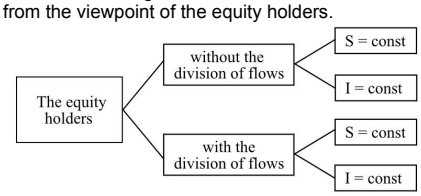
Modigliani and Miller (1958) have created a theory of WACC for perpetuitve companies. For companies with a finite lifetime Brusov and Filatova (2010) have developed a consistent theory of the weighted average cost of capital, WACC.

Table 1. Scheme of address of the problem of the influence of the debt financing on the effectiveness of the investment project from the viewpoint of the owners of equity and debt.



Source: Authors

Table 2. Scheme of address of the problem of the influence of the degree of debt financing on the effectiveness of the investment project from the viewpoint of the equity holders.



Source: Authors

In this paper the real quantitative results were obtained within the Modigliani-Miller theory for the first time. The effectiveness of the investment project is considered from two perspectives: the owners of equity and debt as well as the equity holders only. For each of these cases, NPV is calculated in two ways: with the division of credit and investment flows (and thus discounting the payments at two different rates) and without such a division (in this case, both flows are discounted at the same rate as which, obviously, can be chosen WACC). For each of the four situations two cases are considered: 1) a constant value of equity S; 2) a constant value of the total invested capital I = S + D (D-value of borrowed funds).

2. Assumptions

As it was stated above, the effectiveness of the investment project is considered from two perspectives: the owners of equity and debt as well as equity holders only. In the first case, the interest and duty paid by owners of equity (negative flows), returned to the project because they are exactly equal to the flow (positive), obtained by owners of debt capital. The only effect of leverage in this case – the effect of the tax shield generated from the tax relief: interest on the loan are entirely included into the cost and, thus, reduce the tax base. After tax flow of capital for each period in this case is

$$NOI\ (1-t)+k_d Dt \quad , \tag{3}$$

and value of investment at the initial time moment $T = 0$ is equal to *–I=–S–D*.

Here *NOI*–Net Operating Income (before taxes).

In the second case, investments at the initial time moment are equal to –S, and the flow of capital for the period (in addition to the tax shields $k_d Dt$ it includes a payment of interest on a loan $-k_d D$)

$$(NOI - k_d D)(1-t) \tag{4}$$

Here, for simplicity, we suppose that interest on the loan will be paid in equal shares during all periods. Note, that principal repayment is made at the end of last period.

Some variety of repayment of long–term loans will be considered by us in subsequent articles.

We will consider two different ways of discounting.

1. Operating and financial flows are not separated and both are discounted at the general rate (as which, obviously, can be selected the weighted average cost of capital, WACC). The Modigliani–Miller formula (Modigliani and Miller (1963)) for WACC for perpetuitive projects will be used.
2. Operating and financial flows are separated and are discounted at different rates: the operating flow at the rate equal to the cost of equity, depending on leverage, k_e , and credit flow – at the rate equal to the cost of debt, k_d , which until fairly large values of leverage remains constant and starts to grow only at high values of leverage L, when there is a danger of bankruptcy.

3. The effectiveness of the investment project from the owners of equity and debt point of view.

3.1. Consideration with the division of credit and investment flows

$$NPV = -I + \sum_{i=1}^{n} \frac{NOI(1-t)}{(1+k_e)^i} + \sum_{i=1}^{n} \frac{k_d Dt}{(1+k_d)^i} =$$
$$= -I + \frac{NOI(1-t)}{k_e}\left(1 - \frac{1}{(1+k_e)^n}\right) + Dt\left(1 - \frac{1}{(1+k_d)^n}\right) \tag{5}$$

In perpetuitive limit ($n \to \infty$) (Modigliani–Miller limit) we have

$$NPV = -I + \frac{NOI(1-t)}{k_e} + Dt \tag{6}$$

3.1.1. Case of a constant value of the total invested capital (*I = const*)

Taking into account $D = IL/(1+L)$, one gets

$$NPV = -I\left(1 - t\frac{L}{1+L}\right) + \frac{NOI\ (1-t)}{k_e} \tag{7}$$

For the cost of equity, k_e, and the weighted average cost of capital, WACC, in the theory of Modigliani and Miller (Modigliani and Miller (1963)) we have respectively

$$k_e = k_0 + (k_0 - k_d)L(1-t) \tag{8}$$
$$WACC = k_0(1 - w_d t)$$

$$= k_0(1 - Lt/(1+L)) \tag{9}$$

Substituting (8) into (7), we get

$$NPV = -I\left(1 - t\frac{L}{1+L}\right) + \frac{NOI\ (1-t)}{k_0 + (k_0 - k_d)L(1-t)} \tag{10}$$

Thus, for limiting cases $L = 0$ and $L = \infty$ one has respectively

$$NPV\ (0) = -I + \frac{NOI\ (1-t)}{k_0}, \quad NPV(\infty) = -I(1-t) \tag{11}$$

$$\Delta NPV = NPV\ (\infty) - NPV\ (0) = It - \frac{NOI\ (1-t)}{k_0} \tag{12}$$

.

NPV increases or decreases with leverage depending on the relation between the parameters It and $\frac{NOI(1-t)}{k_0}$. At $It > \frac{NOI\ (1-t)}{k_0}$ *NPV* increases with leverage from $-I + \frac{NOI\ (1-t)}{k_0}$ up to $-I(1-t)$ (curve I at fig. 1.).

At $It < \frac{NOI(1-t)}{k_0}$ *NPV* decreases with leverage from $-I + \frac{NOI\ (1-t)}{k_0}$ up to $-I(1-t)$ (curve II at fig. 1.)

Inequality $It > \frac{NOI\ (1-t)}{k_0}$ can be written. Since we consider the perpetual Modigliani-Miller limit, the last formula is a kind of analogue of the formula for perpetual annuity present value

Inequality $It > \frac{NOI(1-t)}{k_0}$ can be written as $I > \frac{NOI\ (1-t)}{k_0 t}$.

Since we consider the perpetual Modigliani-Miller limit, the last formula is a kind of analogue of the formula for perpetual annuity present value $A > \frac{R}{i}$, where the value of the initial investment plays the role of the eternal present value of rent, net operating income (before taxes) *NOI* serves as the rental payment (DC) and the role played by *i* is the "interest" rate $i = \frac{k_0 t}{(1-t)}$.

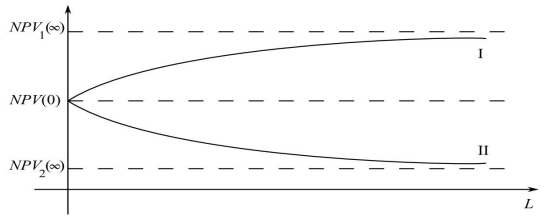
NPV increases with leverage, if the "interest" rate $i > \frac{NOI}{I}$, i.e. if the "rate" *i* is greater than the ratio of net operating income (before taxes) to the initial investment, and decreases with leverage, if it is less than this ratio.

Consider an example. Suppose that investment is *I* = \$2000000. Income tax rate $t =$ 50%, $NOI = \$45000$, $k_0 = 10\%$. Find the dependence of *NPV* on leverage.

Here $i = \frac{k_0 t}{(1-t)} = \frac{0{,}1 \cdot 0{,}5}{0{,}5} = 0{,}1$, and the share of net operating income $\frac{NOI}{I} = \frac{45}{2000} = 0{,}0225$, therefore NPV increases with leverage, because "rate" $i > \frac{NOI}{I}$.

It is easy to see, that at $NOI > \$200000$ *NPV decreases with leverage.*

Fig.1. The dependence of the NPV of the project on the leverage in the case of a constant value of investment (*I* = const).



3.1.2. Case of a constant value of equity S = const

Accounting $D = LS$, $I = S(1+L)$, one gets

$$NPV = -S - LS + \frac{NOI(1-t)}{k_e}\left(1 - \frac{1}{(1+k_e)^n}\right) + Dt\left(1 - \frac{1}{(1+k_d)^n}\right) =$$

$$= -S\left[1 + L - tL\left(1 - \frac{1}{(1+k_d)^n}\right)\right] + \frac{NOI\,(1-t)}{k_e}\left(1 - \frac{1}{(1+k_e)^n}\right)$$

(13)

In perpetual limit ($n \to \infty$) (Modigliani–Miller limit) one has

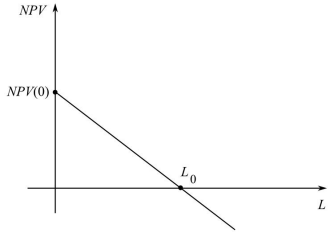
$$NPV = -S(1 + L(1-t)) + \frac{NOI\,(1-t)}{k_0 + (k_0 - k_d)Lt} \qquad (14)$$

Thus, for limiting cases $L = 0$ and $L = \infty$ one has respectively

$NPV\ (0) = -S + \frac{NOI\ (1-t)}{k_0}$, $NPV\ (\infty) = -\infty$. (15)

NPV decreases with leverage from $-S + \frac{NOI\,(1-t)}{k_0}$ up to $-\infty$ (Fig. 2), becoming zero at $L = L_0$.

Fig.2. The dependence of the NPV of the project on the leverage in the case of a constant value of equity (S = const). L_0 – the maximum leverage value, at which the project is still effective (NPV>0).



Note, that in case *S* = *const*, *NOI* is no longer a constant and is generally proportional to the magnitude of investments $NOI = \beta I = \beta S(1+L)$.

In this case formula (14) is substituted by

$$NPV = -S(1 + L(1-t)) + \frac{\beta S(1+L)(1-t)}{k_0 + (k_0 - k_d)Lt} \qquad (16)$$

and $L_0$is determined from the quadratic equation

$$S(1 + L(1-t)) = \frac{\beta S(1+L)(1-t)}{k_0 + (k_0 - k_d)Lt} \qquad (17)$$

Decrease of *NPV* with leverage in this case is due to the fact that the increase in borrowing (negative flow) is not offset by increased *NOI* (positive flow), which grows just from $\frac{\beta S(1-t)}{k_0}$ up to $\frac{\beta S(1-t)}{(k_0 - k_d)t}$.

3.2. Consideration without the division of credit and investment flows

$$NPV = -I + \sum_{i=1}^{n} \frac{NOI\,(1-t) + k_d Dt}{(1 + WACC)^i} =$$

$$= -I + \frac{NOI\,(1-t) + k_d Dt}{WACC}\left(1 - \frac{1}{(1+WACC)^n}\right) \qquad (18)$$

In perpetuitive limit ($n \to \infty$) (Modigliani–Miller limit) one has

$$NPV = -I + \frac{NOI\ (1-t) + k_d Dt}{WACC} \qquad (19)$$

3.2.1. Case of a constant value of the total invested capital (*I = const*)

$$NPV = -I + \frac{NOI\,(1-t) + k_d Dt}{WACC} =$$

$$= -I\left(1 - \frac{k_d t \frac{L}{1+L}}{k_0\left(1 - \frac{L}{1+L}t\right)}\right) + \frac{NOI\,(1-t)}{k_0\left(1 - \frac{L}{1+L}t\right)}. \qquad (20)$$

Thus, for limiting cases $L = 0$ and $L = \infty$ one has respectively

$$NPV\ (0) = -I + \frac{NOI\,(1-t)}{k_0},$$

$$NPV\ (\infty) = -I\,\frac{k_0(1-t) - tk_d}{k_0(1-t)} + \frac{NOI}{k_0} \qquad (21)$$

$$\Delta NPV = NPV(\infty) - NPV(0) = \frac{Itk_d}{k_0(1-t)} + \frac{NOI \cdot t}{k_0} > 0. \qquad (22)$$

This means, that *NPV* grows with leverage from $-I + \frac{NOI\,(1-t)}{k_0}$ up to $-I\,\frac{k_0(1-t) - tk_d}{k_0(1-t)} + \frac{NOI}{k_0}$ (curve I at Fig. 1.).

3.2.2. Case of a constant value of equity (*S = const*)

$$NPV = -I + \frac{NOI\,(1-t) + k_d Dt}{WACC}\left(1 - \frac{1}{(1+WACC)^n}\right) =$$

$$= -S\left[1 + L - \frac{k_d Lt}{WACC}\left(1 - \frac{1}{(1+WACC)^n}\right)\right] +$$

$$+ \frac{NOI\,(1-t)}{WACC}\left(1 - \frac{1}{(1+WACC)^n}\right). \qquad (23)$$

In perpetual limit ($n \to \infty$) (Modigliani–Miller limit) one has

$$NPV = -S\left[1 + L - \frac{k_d Lt}{WACC}\right] + \frac{NOI\,(1-t)}{WACC} \qquad (24)$$

$$NPV = -S\left[1+L-\frac{k_d L t}{k_0\left(1-\frac{L}{1+L}t\right)}\right]+\frac{\beta S(1+L)(1-t)}{k_0\left(1-\frac{L}{1+L}t\right)} \quad (25)$$

Thus, for limiting cases $L=0$ and $L=\infty$ one has respectively

$$NPV(0) = -S+\frac{\beta S(1-t)}{k_0},$$

$$NPV(\infty) = \begin{cases} -\infty, & -k_0(1-t)+k_d t+\beta(1-t)<0, \\ \infty, & -k_0(1-t)+k_d t+\beta(1-t)>0. \end{cases} \quad (26)$$

If *NPV* decreases with leverage, it becomes zero at $L = L_0$

$$L_0 = \frac{k_0-\beta(1-t)}{-k_0(1-t)+k_d t+\beta(1-t)} \quad (27)$$

Note, that $L = L_0$ is the maximum leverage value, at which the project is still effective (*NPV*>0).

4. The effectiveness of the investment project from the equity holders viewpoint.

4.1. Consideration with the division of credit and investment flows

$$NPV = -S+\sum_{i=1}^{n}\frac{NOI(1-t)}{(1+k_e)^i}+\sum_{i=1}^{n}\frac{-k_d D(1-t)}{(1+k_d)^i} =$$

$$= -S+\frac{NOI(1-t)}{k_e}\left(1-\frac{1}{(1+k_e)^n}\right)-\frac{Dk_d(1-t)}{k_d}\left(1-\frac{1}{(1+k_d)^n}\right) \quad (28)$$

In perpetuitive limit ($n\to\infty$) (Modigliani–Miller limit) one has

$$NPV = -S+\frac{NOI(1-t)}{k_e}-\frac{Dk_d(1-t)}{k_d} \quad (29)$$

4.1.1. Case of a constant value of the total invested capital (*I* = *const*)

Taking into account $D = IL/(1+L)$, $S = I/(1+L)$, we get

$$NPV = -\frac{I}{1+L}\left(1+\frac{Lk_d(1-t)}{k_d}\right)+\frac{NOI(1-t)}{k_e} \quad (30)$$

$$NPV = -\frac{I}{1+L}\left(1+\frac{Lk_d(1-t)}{k_d}\right)+\frac{NOI(1-t)}{k_0+(k_0-k_d)L(1-t)} \quad (31)$$

Thus, for limiting cases $L = 0$ and $L=\infty$ one has respectively

$$NPV(0) = -I+\frac{NOI(1-t)}{k_0}, NPV(\infty) = -I(1-t) \quad (32)$$

$$\Delta NPV = NPV(\infty)-NPV(0) = It-\frac{NOI(1-t)}{k_0} \quad (33)$$

As estimations show, *NPV* decreases with leverage (curve II at Fig. 1).

4.1.2. Case of a constant value of equity (*S* = *const*)

Accounting $D = LS$, in perpetual limit ($n\to\infty$) (Modigliani–Miller limit) one has

$$NPV = -S\left[1+\frac{Lk_d(1-t)}{k_d}\right]+\frac{NOI(1-t)}{k_e} =$$

$$= -S(1+L(1-t))+\frac{\beta S(1+L)(1-t)}{k_0+(k_0-k_d)L(1-t)} \quad (34)$$

In perpetuitive limit ($n\to\infty$) (Modigliani–Miller limit) one has

$$NPV(0) = -S+\frac{\beta S(1-t)}{k_0},\ NPV(\infty) = -\infty.$$

So, *NPV* decreases with leverage from $-S+\frac{S\beta(1-t)}{k_0}$ in the absence of debt financing up to $-\infty$ (Fig.2), becoming zero at $L = L_0$, determining from the quadratic equation

$$S(1+L(1-t)) = \frac{S\beta(1+L)(1-t)}{k_0+(k_0-k_d)Lt} \quad (35)$$

4.2. Consideration without the division of credit and investment flows

$$NPV = -S+\sum_{i=1}^{n}\frac{NOI(1-t)-D(k_d(1-t))}{(1+WACC)^i} =$$

$$= -S+\frac{NOI(1-t)-Dk_d(1-t)}{WACC}\left(1-\frac{1}{(1+WACC)^n}\right) \quad (36)$$

In perpetuitive limit ($n\to\infty$) (Modigliani–Miller limit) one has

$$NPV = -S+\frac{NOI(1-t)-Dk_d(1-t)}{WACC}. \quad (37)$$

4.2.1. Case of a constant value of the total invested capital (*I* = *const*)

Taking into account $D = IL/(1+L)$, $S = I/(1+L)$, we get

$$NPV = -I\cdot\frac{1}{1+L}+\frac{NOI(1-t)-I\frac{Lk_d(1-t)}{1+L}}{WACC} =$$

$$= -I\cdot\frac{1}{1+L}\left[1+\frac{Lk_d(1-t)}{k_0(1-Lt/(1+L))}\right]+\frac{NOI(1-t)}{k_0(1-Lt/(1+L))} \quad (38)$$

Thus, for limiting cases $L = 0$ and $L=\infty$ one has respectively

$$NPV(0) = -I+\frac{NOI(1-t)}{k_0} \quad (39)$$

$$NPV(\infty) = -I\left[\frac{k_d}{k_0}\right]+\frac{NOI}{k_0} \quad (40)$$

$$\Delta NPV = NPV(\infty)-NPV(0) = -I\left[\frac{(k_d-k_0)}{k_0}\right]+\frac{NOI\cdot t}{k_0}$$

As estimations show, *NPV* increases with leverage (curve I at Fig. 1).

4.2.2. Case of a constant value of equity (*S* = *const*)

$$NPV = -S+\frac{NOI(1-t)-Dk_d(1-t)}{WACC} \quad (41)$$

Substituting $D = LS$, we get

$$NPV = -S\left[1 + \frac{Lk_d(1-t)}{WACC}\right] + \frac{NOI(1-t)}{WACC} =$$

$$= -S\left[1 + \frac{Lk_d(1-t)}{k_0(1-Lt/(1+L))}\right] + \frac{\beta S(1+L)(1-t)}{k_0(1-Lt/(1+L))} \quad (42)$$

Thus, for limiting cases $L = 0$ and $L = \infty$ one has respectively

$$NPV(0) = -S + \frac{S\beta(1-t)}{k_0} \quad (43)$$

$$NPV(\infty) = \begin{cases} -\infty, & (k_d - \beta)(1-t) > 0, \\ \infty, & (k_d - \beta)(1-t) < 0. \end{cases} \quad (44)$$

Thus, at $(k_d - \beta)(1-t) > 0$ *NPV* decreases with leverage from $-S + \frac{S\beta(1-t)}{k_0}$ up to $-\infty$, becoming zero at $L = L_0$, determining from the equation

$$k_0(1 - Lt/(1+L)) + L(k_d(1-t)) = \beta(1+L)(1-t) \quad (45)$$

At $(k_d - \beta)(1-t) > 0$ *NPV* increases with leverage from $-S + \frac{S\beta(1-t)}{k_0}$ up to ∞.

CONCLUSION

In this paper for the first time during many years of research of the problem of the influence of debt financing on the effectiveness of the investment project tangible results have been obtained within the theory of Modigliani–Miller.

The effectiveness of the investment project is considered from two perspectives: the owners of equity and debt and equity holders only. It was shown, that within the theory of Modigliani–Miller *NPV* practically always decreases with leverage (*L=D/S*) in case of a constant value of equity (*S=const*). For each of four cases (at *S=const*) the maximum leverage level, at which the project is still effective (*NPV*>0), was found.

In case of a constant value of the total invested capital (*I=const*) it is possible an increase of *NPV* with leverage (unlimited as well as in the saturation regime, i.e. *NPV* asymptotically reaches a maximum value at infinite leverage), as well as decrease of *NPV*: it depends on the relation between the parameters of the project $(NOI, k_e, k_d, t, \beta)$. The conditions for increasing of *NPV* with leverage are formulated. All the obtained dependencies of *NPV*(*L*) are monotonic, which means the absence of optimal leverage in the theory of Modigliani–Miller. This theory is basic in the sense that it can easily be adapted to different realities of the investment project, for example, different schemes of payment of interest on the loan, different schemes of principal repayment and other conditions.

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