MICROSCOPIC MODEL OF KNOWLEDGE INCREASE AND ITS VERIFICATION

Simona Dudáková[1], Michal Benko[2], Boris Lacsný[3], Aba Teleki[4]

Abstract: Knowledge growth models, based on primary principles, play a fundamental role in the cognitive sciences. The authors submit an extension of their model (ENKI) from 2005, with the results of the practical testing, which was performed using the method developed for the purpose of model ENKI of solving tasks with immediate feedback. This was applied to the curriculum of parallel configuration of resistors in electrical circuits. There were 73 pupils from six elementary schools in attendance for testing. Analysis based on ENKI indicates that three autonomous units (scopes) were evaluated simultaneously during the assessment. Results showed that 25% of pupils knew the curriculum, 9% of pupils showed no improvement, while 66% of pupils showed an increased success in accordance with the ENKI model (significance level $\alpha = 0.05$). Solving 7.2 typical tasks on average, by a method of immediate feedback resulted in 90% of the pupils mastering the curriculum.

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Introduction

Neuroscience provides an insight into the formation of cognitive structures in the brain from a biological point of view (Fields, 2005). Research indicates that repetition creates long-lasting neural connections in the brain in processes which are random in essence. If the created structure of connections is correct it allows the person to use the new knowledge in practice with success. The latest research indicates that connections with autonomous functionality may also be a single neural connection (Quiroga, Kraskov, Koch, & Fried, 2009). Many empirical mathematical models of the knowledge build-up are known to date. Simple models were built, for example, by Hickling (1976), Preece (1984), and Anderson (1983). The scientific description of knowledge or growth of knowledge is crucial to gain more insight into the understanding of experimental learning. There are numerous approaches to implement and analyze trials, ranging from the use of standardized tests for gain factor measurement (Bao, 2006; Meltzer, 2002), to international studies of large groups of people (Bao et al., 2009). The Efficacy Norm Increase of Knowledge (ENKI) model is a unique approach which comes from the probability description of neural connections growing in the brain during the process of learning.

The ENKI Model

In the process of learning, unused synapses of a neuron are activated by firing an impulse through them. The synapses are activated and short-term memory forms. The short-term memory formed by the firing is converted into long-term memory by repetition in a process controlled by the nucleus of the neuron. This is a random process in the sense that there is no mechanism addressing a sole activated synapse between hundreds of non-activated synapses of the neuron (Fields, 2005).

The probability of converting the short-term memory to long-term memory, accomplished by the activated synapse, by one repetition is p. If no repetition, is done the activated state of the synapse diminishes back to its original (non-activated) state within a few hours. This temporary synaptic strengthening is a cellular model of short-term memory.

Successful actions are repeated in the process of learning. On the one hand, repeated actions firing signals across the same synapses elongate the activated state of the synapses. On the other hand, the nucleus of the neuron is stimulated to repeat the process of converting short-term memory into long-term memory. This is how long-lasting synapses are formed in repeated random processes. On this basis, the model ENKI was constructed (Lacsný, 2005).

[1] Simona Dudáková, Department of Physics, Constantine the Philosopher University, Nitra, Slovakia, simona.dudakova@ukf.sk
[2] Michal Benko, Department of Physics, Constantine the Philosopher University, Nitra, Slovakia, benko.michal@centrum.sk
[3] Boris Lacsný, Department of Physics, Constantine the Philosopher University, Nitra, Slovakia, blacsny@ukf.sk
[4] Aba Teleki, Department of Physics, Constantine the Philosopher University, Slovakia, ateleki@ukf.sk

This model has a significant assumption that the successful action needs repetition in a specific period of time, as defined by chemical processes in neural nuclei (Fields, 2005). Every succeeding repetition needs some delay between repetitions that is not too short and not too long. The number of repetitions in a successful learning process is proportional to time – the duration of the learning process. When the formation of the required neural structure is interrupted for an unnaturally long time, random and spontaneous processes degrade the activated (but unstable) neural structures (Lacsný, 2005).

The concept that knowledge is growing exponentially in time is generally accepted in most phenomenological models (Gamble, 1986; Hassan, 2005; Pritchard, Lee, & Bao, 2008). In ENKI, this property is a consequence of principles stated in Fields (2005). ENKI was refined on the basis of research in physical problem solving from 2005 to date (Lacsný, 2005; Benko, 2013; Dudakova, 2016).

In its simplest form, ENKI predicts the probability $P(n)$ of achieving success in solving the n-th problem in a sequence of similar problems. In the sequence of similar problems there must be at least one common feature; a particular, well-defined task that must occur in each problem of the sequence. Such a task may be to construct a two-dimensional cartesian frame, to find the roots of a second order equation, to make a decision about whether an image shows a parallel or serial circuit, or something else meaningful from the viewpoint of evaluability. In more detail, ENKI predicts only the probability $P(n)$ of achieving success in solving these common tasks occurring in the sequence of problems.

The probability $P(n)$ has the form

$$P(n) = (1 - q^n)^{N_a} \,, \tag{1}$$

where N_a is the number of autonomous units in the neural sub-network responding to the ability to correctly solve the observed common task of the sequence of problems. In this instance, N_a is assumed to be the number of synapses in the neural sub-network. Then $q = 1 - p$ is the probability that an arbitrary but concrete synapse of the neural sub-network is not established by solving the common task in the sequence of problems. The model assumes that q is the same for all synapses and that its value is not dependent on time; more precisely, not dependent on the number of repetitions.

The ENKI model is language independent and is not task specific. The model is capable of fitting the recollection growth or knowledge growth for various complexity of tasks (Benko, 2013), but, in its current form, is a probabilistic model of learning of one individual.

In accordance with this limitation, we observed during real testing that not every participant achieved an increase of knowledge. To verify the model, physics problem solving abilities were analyzed in primary and secondary schools, and also universities. We present results obtained in the primary schools only. However, these are very similar (from the viewpoint of practicality of ENKI) to other results not yet published.

On one hand, some of the pupils were unable or had no leaning to achieve success in testing, on the other hand, some had pre-requisite knowledge prior to testing.

For this purpose, the model prescription was extended by the parameters, $\nu^{(r)}, \nu^{(p)}, and\ \nu^{(f)}$. Here $\nu^{(r)}$ was the ratio of pupils who possessed the required knowledge and were ready to use this knowledge (group G_r). The ratio $\nu^{(p)}$ denoted pupils who achieved progress in problem solving (i.e., students who had growth in knowledge, group G_p). The ratio $\nu^{(f)}$ of pupils who failed in solving the problem sequence was then $\nu^{(f)} = 1 - \nu^{(r)} - \nu^{(p)}$ (group G_f). After this modification, the probability $P(n)$ had the form

$$P(n) = \nu^{(r)} + \nu^{(p)}(1 - q^n)^{N_a} \tag{2}$$

The new parameters allowed us to quantify some initial conditions influencing the trials.

This modification of the ENKI model also appeared to be a powerful tool in assessing the quality of the course, and also the motivation of pupils in the course as we show below.

The Trial in Primary Schools

The trial, presented here, was accomplished in 2013 in primary schools with $N = 73$ pupils participating. Testing was prepared for pupils aged from 14 to 15 years. The exercise was to calculate the equivalent electrical resistance of two or more resistors connected in parallel. The pupils had passed the curriculum three months before the trial.

The testing methodology consisted of: (1) review and interpret the curriculum, using multimedia presentation instead of the standard textbook (Stelzer, Gladding, Mestre, & Brooks, 2009); (2) the first of ten physics problems being presented to the pupils; and (3) after the task finished, the pupils were given feedback on the solution, and this involved the correctness of solutions and the entire solution method necessary for use. Steps (2) and (3) were repeated for the remaining nine physics problems.

When the pupils solved problems incorrectly, and the mistake was one of the most frequent (expected), feedback included information about that mistake and how it should be avoided in subsequent tasks.

Each task was evaluated as “1” in a case of correct answer and “0” in any other case. The obtained data were averaged (s_n) and plotted to show relative efficiency depending on the number of repetitions (solved physics problems).

Results

We estimated parameters $q, N_a, \nu^{(r)}$, and $\nu^{(p)}$ in Equation (2) by minimizing χ^2, as follows:

$$\chi^2 = \sum_{n=1}^{N} N^2 \frac{\left(s_n - P(n)\right)^2}{\sigma_n^2} \tag{3}$$

where s_n was the average score of pupils for the n-th task in the mentioned sequence of tasks ($N = 10$ physics problem) and σ_n^2 the variance of scores in the n-th task.

We assumed that data for a fixed n (number of repetition) had binomial distribution ($\sigma_n^2 = N\, P(n)\big(1 - P(n)\big)$, but also normal distribution was appropriate for data near the theoretical values given by Equation 2.

We found the following values for the parameters and confidence intervals at the significance level $\alpha = 0.05$: $p = (0.653 \pm 0.019)$, $N_a = (1.61 \pm 0.18)$, $\nu^{(r)} = (0.199 \pm 0.013)$, and $\nu^{(p)} = (0.702 \pm 0.016)$. The experimental value of $\chi^2 = 2.176$ and the χ^2 test failed at the significance level $\alpha_{\max} = 0.35$.

To avoid problems with the 4-dimensional domain of confidence, we assumed that all parameters ($\nu^{(.)}, q, and\ N_a$) contributed to the uncertainty at the same rate (with the same probability). Despite pupils having already studied the curriculum three months prior, only about 20% ($\nu^{(r)} = 0.199$) of the students were able to solve the problems correctly from the beginning of the assessment. About 70% ($\nu^{(p)} = 0.702$) of them indicated progress during the assessment and about 10% had no success or motivation to succeed during the assessment. To obtain more information about circumstances influencing the growth of knowledge during the assessment, we discuss the meaning of the autonomous units in the ENKI model in the following.

The current form of the ENKI model was based on the microscopic background of the learning process. The results obtained by analyzing data were interesting from two points. First, N_a was not an integer with $N_a = (1.61\ {}^{+0.20}_{-0.18})$. Second, N_a had a very low value (1.61). In this version of the model, N_a, was the number of synapses necessary to achieve success in solving the given problem and these synapses were established independently.

The low value of N_a may indicate two possibilities, not excluding of each other: (1) the ability to solve a new problem may be connected to only one synapse (connecting to the existing “library”' of long-lasting synapses), and (2) an autonomous unit is a set of synapses that fire simultaneously during the problem solving.

In either case, autonomous units develop independently during the learning process. In the frame of ENKI that means that autonomous units may be built up individually.

It may be, that the brain has its "built-in" algorithm to divide any problem into smaller pieces. These pieces have their own history of learning and each their own background consisting of other correct working pieces, a library.

We show below, using the ENKI model, that the number of autonomous units is insignificant, though greater than one and most probably equal to three, when the task involves calculating the equivalent electrical resistance of two or more resistors connected in parallel.

The non-integer property may indicate, in our opinion, the following features influencing the results of an assessment: (1) the group of pupils achieving progress during the assessment (group G_p) was composed of groups with different learning histories. They did not have all autonomous units or the library required for solving the tasks; (2) autonomous units consisted of a greater number of synapses and therefore all autonomous units had their own parameter, p; and (3) all synapses operated with a different parameter, p.

We analyzed the most probable cases.

If the required sub-network consisted of two autonomous units A_1 and A_2, the most general form of the probability $P(n)$ was

$$P(n) = \nu^{(r)} + \nu_1^{(p)}(1 - q_1^n) + \nu_2^{(p)}(1 - q_2^n) + \nu_{12}^{(p)}(1 - q_1^n)(1 - q_2^n) \quad (4)$$

where the second ν term describes the subgroup $G_{p,1}$ of pupils with the created and operating A_2 but not A_1. The third term describes the subgroup $G_{p,2}$ of pupils with the created and operating A_1 but not A_2. The last term describes the subgroup $G_{p,12}$ of pupils who had not created A_1 nor A_2.

We found the following values for the parameters and confidence intervals at the significance level $\alpha = 0.05$: $q_1 = (0.708\ ^{+0.017}_{-0.010})$, $q_2 = (0.484\ ^{+0.089}_{-0.063})$, $\nu^{(r)} = (0.222\ ^{+0.010}_{-0.006})$, $\nu_1^{(p)} = (0.00\ ^{+0.012}_{-0.000})$, $\nu_2^{(p)} = (0.00\ ^{+0.11}_{-0.00})$, $and\ \nu_{12}^{(p)} = (0.00\ ^{+0.71}_{-0.01})$. The experimental value of $\chi^2 = 2.168$ and the χ^2 test failed at the significance level $\alpha_{\max} = 0.08$.

The results show that in Equation 4, one can exclude the second and third term before the minimization procedure (greater number of freedoms).

Considering three autonomous units A_1, A_2, and A_3 we have several possibilities that cannot be analyzed in one step due to the large number of parameters to estimate. The same is true for larger number of autonomous units. Analyzing, step by step, we found the most reliable structures, as follows:

$$P_{12}(n) = \nu^{(r)} + \nu_{12}^{(p)}(1 - q_1^n)(1 - q_2^n) \quad (5)$$

$$P_{123}(n) = \nu^{(r)} + \nu_{123}^{(p)}(1 - q_1^n)(1 - q_2^n)(1 - q_3^n) \quad (6)$$

$$P_{1234}(n) = \nu^{(r)} + \nu_{1234}^{(p)}(1 - q_1^n)(1 - q_2^n)(1 - q_3^n)(1 - q_4^n) \quad (7)$$

The dependence of every $P(n)$ is just the same with no significant difference and therefore also the experimental value of χ^2 is just the same ($\chi^2_{12} = 2.168$, $\chi^2_{123} = 2.148$, $and\ \chi^2_{123} = 2.132$).

Most important is the level of significance at which the χ^2 test failed. Using the same notation, we found that the χ^2 test failed at level $\alpha_{12} = 0.349$, $\alpha_{123} = 0.417$, and $\alpha_{1234} = 0.09$.

We can see, that Eq. (5) and (6) are competitive structures. Analyzing the structure, as follows:

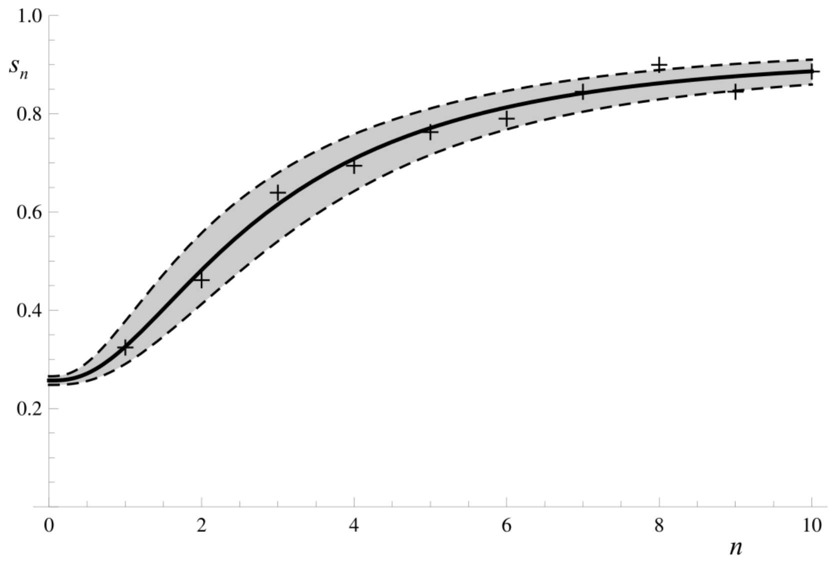
$$P(n) = \nu^{(r)} + \nu_{12}^{(p)}(1 - q_1^n)(1 - q_2^n) + \nu_{123}^{(p)}(1 - q_1^n)(1 - q_2^n)(1 - q_3^n) \quad (8)$$

we obtain $\nu_{12}^{(p)} = 0.000\ ^{+0.008}_{-0.0}$.

After all we can conclude that the most reliable description of the results of the assessment is given by Equation 6 with the values of estimated parameters $q_1 = (0.708\ ^{+0.014}_{-0.015})$, $q_2 = (0.457\ ^{+0.111}_{-0.145})$, $q_3 = (0.302\ ^{+0.118}_{-0.142})$, $\nu^{(r)} = (0.257 \pm 0.009)$, and $\nu_{123}^{(p)} = (0.655 \pm 0.011)$.

The result is shown in Figure 1.

Figure 1: Experimental data (averaged score s_n) fitted by ENKI model (solid curve) and confidence bands on the level of significance $\alpha = 0.05$ (filled area between dashed curves). The values of estimated parameters are $q_1 = (0.708\ {}^{+0.014}_{-0.015})$, $q_2 = (0.457\ {}^{+0.111}_{-0.145})$, $q_3 = (0.302\ {}^{+0.118}_{-0.142})$, $\nu^{(r)} = (0.257 \pm 0.009)$, $\nu^{(p)}_{123} = (0.655 \pm 0.011)$. The confidence band shows the theoretical domain with 95 % of the most important data. The narrowness of the band also confirms the reliability of analysis using the ENKI model.



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Final Remarks Concerning the Data Analysis

We averaged the answers of the 73 pupils for each task of the assessment. Hence, 10 pieces of averaged data were analyzed using the model ENKI and confidence intervals were relatively wide. We aimed to avoid possible ambiguities in our analysis by comparing our results with those obtained using randomly generated data that had Bernoulli distribution: $\binom{N}{k} p^k q^{(N-k)}$, where $N = 73$ was the number of pupils in the assessment, k the number of correct answers, and $p = P(n)$ and $q = 1 - P(n)$ the probabilities given by ENKI. We found that our standard analysis was sufficiently sound and capable of finding the correct values of estimated parameters within the confidence interval.

1) The ENKI model is a non-linear model for the most part (it is linear in parameters ν only), which gives the probability $P(n)$ in the form

$$P(n) = \sum_I P_I(n) \tag{9}$$

where $I = \{i_1, \dots, i_k\}$ is an index set and $P_I(n)$ is partial probability describing simultaneous evolution of autonomous units $A_{i_1}, \dots, A_{i_k}$, as follows:

$$P_I(n) = \nu_I^{(p)} \prod_{i \in I}(1 - q_i^n)\,; \qquad \nu_I^{(r)} \geq\ 0 \tag{10}$$

and for $I = \emptyset$ we have $\nu_{\emptyset}^{(p)} \equiv \nu^{(r)}$. Every $P_I(n)$ is a monotonic function of n (if $I = \emptyset$, $P_\emptyset = const.$), and data fluctuation cannot be removed by increasing the number of autonomous units in ENKI. In other words, the number of degrees of freedom could have been underestimated in the analysis.

2) On one hand, the average score, s_n, may have achieved its maximal value promptly, and further repetition yielded no further significant contribution to analysis. It is the case also for relatively large number of autonomous units. On the other hand, one cannot implement a large sequence

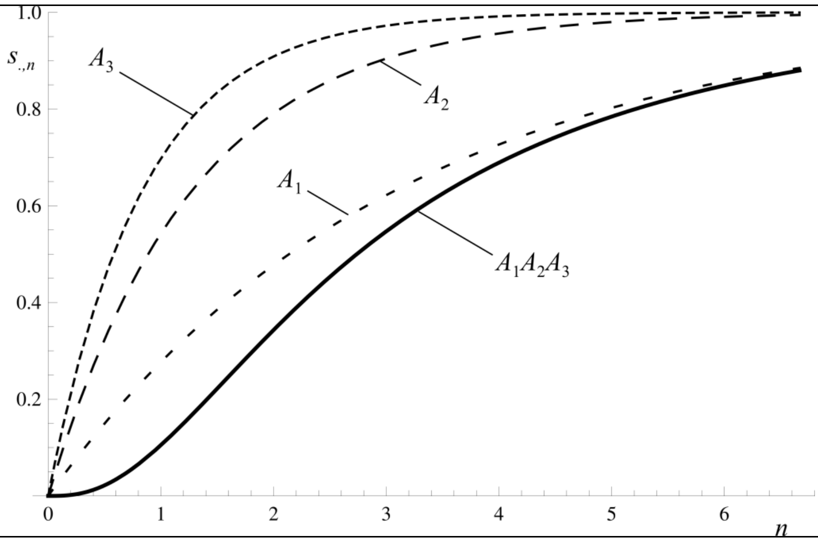
of complicated problems (problems with many autonomous units) in assessment for its time-consuming pattern. (This is not the case if one investigates short-term memory.)

Results and Discussion

We analyzed the growth of knowledge in 73 pupils in primary school during an assessment. Pupils solved ten similar physics problems and explanatory feedback was given immediately after completing a task from the aforementioned sequence of tasks. The exercise was the same in all tasks, to calculate the equivalent electrical resistance of two or more resistors connected in parallel. Pupils had the curriculum three months before the trial. Each score was averaged across the 73 pupils and analyzed using the ENKI model.

1) Our analysis showed that, most probably, three independent scopes were formed in the learning process during the assessment, and this influenced the score of pupils. Using the average scores, we could only deduce the values, but the ENKI model indicated the number of autonomous units in a learning process with high significance. On the other hand, we can find, not only the number of autonomous units playing a significant role in the learning process, but also decipher its evolution during the learning process (Figure 2). This was the benefit of the ENKI model.

Figure 2: The evolution of individual averaged scores $s_{1,n}$, $s_{2,n}$, $s_{3,n}$ and $s_{123,n}$ connected with individual autonomous units A_1, A_2, A_3 (dashed curves) and experimental data fitting by ENKI (solid curves) in group G_p of pupils. Autonomous units A_3 and A_2 are formed soon, while A_1 is more responsible for the evolution at the final stage



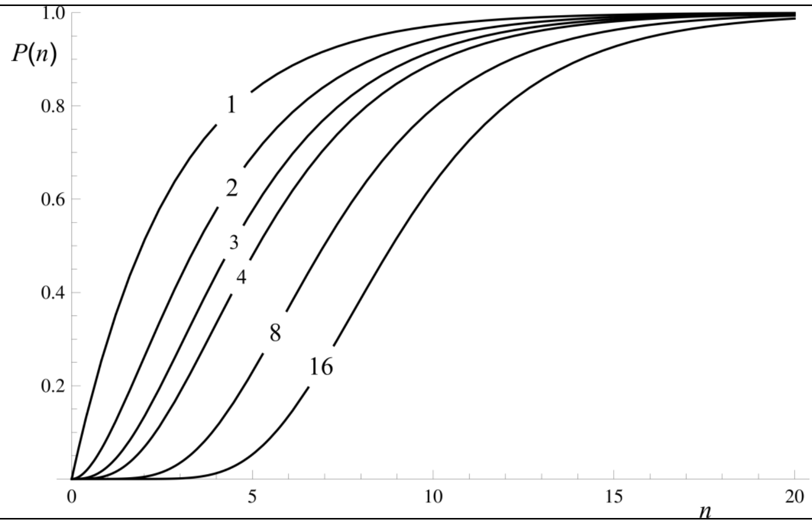
Source: Authors

In much of the present research in physics education, it is considered that knowledge consists of smaller units than the concepts that are discussed in curricula. Hammer (1996, 2000), Minstrell (1982), diSessa and Sherin (1998), diSessa (1988), Redish (1994), and McDermott and Redish (1999) have all alluded to the notion that students do not have well developed or coherent mental models that they use, but rather activate pieces of knowledge. Pupils may understand units differently (Reid & Yang, 2002). A part, expected as a single unit by an expert, could consist of many subunits forming independently in the learning process. However, the number of autonomous units identified by ENKI may return the genuine algorithm of the brain applied during the learning process, rather than our concepts of units (or subunits) in the curricula.

Consequently, it should come as no surprise that students often learn far too little in physics courses, although they may learn more when courses are structured to facilitate better evaluative feedback to students via interactive engagement methods (Cummings, Marx, Thornton, & Kuhl, 1999; Hake, 1998).

2) The typical S-like shape in the score progression occurs when more autonomous units are employed (Figure 3).

Figure 3: The evolution of $P(n)$ for up to 20 repetitions and for different number of autonomous units. We used the simplest form given by Eq. (1) and with $q = 0.7$ for all autonomous units.



Source: Authors

The number of problems solved in a sequence with immediate feedback is crucial to achieving long-lasting knowledge. In our case, 3.8 tasks (on average) were necessary to acquire the required knowledge in the case of 50% of group G_p pupils achieving progress during the assessment. For the same result in the case of 90% of the group G_p an average 7.2 tasks were needed.

This is an important common feature of any learning process, often underestimated by teachers and pupils. Our finding supports the statement that only 25.7% of pupils solved the tasks of the assessment correctly from the beginning despite having passed the curriculum three months prior.

Model ENKI allows for estimation of score progression in any learning process. If the values of parameters are unknown, one can make a rough estimation; for example, 0.5 for q (as an average in our case) and an integer number for the number, N_a, of autonomous units (Equation 1 is ideal for this purpose). The number of autonomous units is at least the number of independent units that are learned in the process. Seeing the "learning curve'' may provide a suitable method for guiding younger teachers with less experience.

3) The ENKI model is capable of supporting the design and evaluation of online learning activities. If, for example, $\nu^{(r)}$ is high, the activity underestimates the level of participants. If $\nu^{(f)}$ is too high, the activity overestimates the level of participants, or their motivation is low. For example, authors of the article performed a trial at a certain technical university with 710 undergraduate students. Our finding was $\nu^{(r)} \approx 0.6$ and $\nu^{(f)} \approx 0.23$. In other words, 60% of students solved correctly the common parts of problems from the beginning of the assessment, but 23% of students achieved no progress during the assessment, with no effect of the instantaneous feedback.

Conclusion

ENKI appears as a trustworthy tool to analyze cognitive processes. Estimated parameters have a good interpretation; therefor ENKI is also an excellent tool for computer simulations of learning processes. Combined with the instantaneous feedback, ENKI presents an integrated learning-assessment procedure.

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