STEADY-STATE OSCILLATIONS OF TWO-BEARING CONSOLE ROTOR WITH MASS IMBALANCE AND DISC MISALIGNMENT

Zharilkassin Iskakov, Almaty University of Energy and Communication, iskakov53@mail.ru

Steady-state oscillations of two-bearing console rotor with mass imbalance and disc misalignment are subject to study. External damping forces can be included into any motion equations. This enables us to generate amplitude-frequency and phase-frequency responses, and to describe rotor behavior at critical speeds. The combined mass imbalance and disc misalignment effect on rotor dynamics can be observed over the entire shaft speed range. Detection of residual phase shift angles at low speeds is practically relevant. This is extremely important for rotor balancing as low-speed measuring findings are used to evaluate mass eccentricity lines. To determine an unknown orientation of disc misalignment line, you can use phase shift angle values at high rotation speeds. The residual phase shift angles are studied depending on combined imbalance and shaft console value. Motion equations and all calculation formulas are given in a compact and dimensionless form.

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Introduction

High-speed rotary machines are known to be widely used in a variety of industries (electric power, electronic and radio engineering, food, light, chemical, oil, medical, metallurgical, space, nuclear industries, etc.). Their high efficiency, small specific weight and high specific power, low production and maintenance costs, as well as low environment pollution result in diversification of rotary machine applications. Hence, it is no wonder that rotary machines have been studied so long. However, there are many unsolved problems, in particular attributable to combined mass imbalance and disc misalignment effect on the oscillations and stability, and other problems related to balancing unbalanced rotary machine parts. Benson (1983) explores linear steady-state oscillations of the unbalanced console rotor. Combined effect of misalignment and disc mass imbalance, i.e. centrifugal effect and active gyroscopic moment effect result in the situation when the processing rotor has an uncommon phase response curve. However the oscillation phase does not necessarily correspond to imbalance orientation at low speeds. This circumstance can change significantly the rotor balancing methods. In view of friction forces inclusion only in the translational rotor motion equations, the resonance amplitudes of translational and angle motions rise unnaturally at second critical speed, which results in a distorted picture of rotor amplitude-frequency and phase-frequency response curves. Grobov (1961) unlike Benson considers unsteady oscillations of the flexible rotor on elastic bearings with nonlinear response, but does not study any dynamics interaction with the generalized disc imbalance. Most studies deal with the definition of position and orientation of mass imbalance, as well

as balancing methods of mass eccentricity. Gordeyev and Maslov (2008) offer acoustic methods related to determination of rotor mass imbalance. Lingener (1991) compares two balancing methods, where the first one is based on measuring an effect on the balancing device bearings, while the other one is based on measuring the respective deformation. Artyunik (1992) uses a surplus balance with several pendulums for automatic balancing. The rotor motion is simulated on a computer. The works of Gordeyev & Maslov (2008), Artyunik (1992) consider various balancing methods for mass imbalance, but do not cover the effect of rotor disc misalignment. Some studies are related to motion stabilization using a control force or a control body. Hendricks & Klauber (1984) demonstrate control capabilities for the entire system using feedback coupling force. The control force has been applied to the second cylinder, placed on a flexible shaft fixed on its two ends at a distance from the first cylinder, partially filled in with perfect incompressible fluid. This system is designed in such a way to provide a close control for the control reasons, where the imbalance source (first cylinder) is associated with the control force (applied to the second cylinder) only with the dynamic connection. This problem can be solved with the help of linear control theory for the rotor-liquid system with its plane model. In spite of the fact that this theory is more attractive, the article does not provide a real mechanism which could be used for the machine control. Tolubayeva (2007) offers practical rotor control methods using connecting-rod mechanism, study theoretical and experimental of hydro elastic oscillations of the vertical gyroscopic rotor, taking into account the energy source impact.

This article covers the external damping in all four motion levels for complete description of the dynamics of two-bearing console rotor with two generalized disc imbalances. Thanks to that, we can describe correctly the amplitude-frequency and phase-frequency rotor response curves, evaluate the impact of mass imbalance and disc misalignment on them, shaft overhanging and external damping, as well as to compare the oscillation amplitudes at critical speeds.

Motion equations of unbalanced rotor in view of the external resistance

Figure 1 shows rotor geometry. The disc is fixed at the end of tail shaft with length a and flexural rigidity EI . The disc has a mass m , a polar moment of inertia I_p and an equatorial moment of inertia I_T . The rotor spins with angular speed ω . Its center of mass is displaced at some distance e from the disc fixing point S with the shaft, and disc itself is tilted slightly τ with respect to the plane and perpendicular shaft axis.

Figure 1: The geometry of the rotor

Source: Author

There is the angle between the line of maximum disc misalignment and mass eccentricity vector β . There is a distance between the bearings l and an external damping χ_e in the bearing next to the console overhang. The shaft with a similar surplus damping can be shown with a surplus bar, which rigidity rate generates viscous bearing damping against the turn (Dimentberg, 1959). Two reference systems are used. System $Oxyz$ is fixed over a distance, while axes x, y enable to identify the position of point S , and rotation axis z passes through the axis of unstrained shaft. We can denote the shaft deviation angles in the planes x, z and y, z by θ_x and θ_y , respectively. Any displacements in x and y directions are considered to be small, and in z direction – neglected. The second system of $SXYZ$ coordinates spins along with the disc, where Z is a polar axis and X axis goes through the mass eccentricity vector.

The rotor motion equations are derived using the Lagrangian method (Yablonskiy 2007; Surin, 2008.). If we use the expressions of disc kinetic energy, potential energy of bent shaft (Grobov, 1961), Rayleigh dissipative function (Yablonskiy, 2007), and put the legends $u = x + iy$, $\theta = \theta_x + i\theta_y$, $L = a + l$ and dimensionless values according to the formulas

$$\begin{aligned} \varepsilon &= e / L, U = u / L, \bar{t} = t(2EI / mL^3)^{1/2}, \Omega = \omega(mL^3 / 2EI)^{1/2}, \\ \bar{I}_p &= I_p / mL^2, \bar{I}_T = I_T / mL^2, \chi = \chi_e(2EI / mL^3)^{1/2}, \\ c_1 &= \frac{1}{(a/L)^3} \frac{1+2(a/L)}{4-(a/L)}, \quad c_2 = \frac{1}{(a/L)^2} \frac{2+(a/L)}{4-(a/L)}, \quad c_3 = \frac{1}{(a/L)} \frac{3}{4-(a/L)} \end{aligned} \quad (1)$$

we can bring the equations of motion to a dimensionless form (θ and τ variables are already dimensionless):

$$U'' + \chi(6c_1U' - 3c_2\theta') + 6c_1U - 3c_2\theta = \varepsilon\Omega^2 e^{i\Omega\bar{t}}, \quad (2)$$

$$\bar{I}_T\theta'' - i\Omega\bar{I}_p\theta' + \chi(-3c_2U' + 2c_3\theta') - 3c_2U + 2c_3\theta = (\bar{I}_p - \bar{I}_T)\varepsilon\Omega^2 e^{i(\Omega\bar{t} + \beta)}. \quad (3)$$

The strokes mean dimensionless time derivatives $\bar{t}$. Bringing the equations of motion to forms (2), and (3) is suitable not only to estimate the oscillation response curves under the influence of mass imbalance and disc misalignment, but also to estimate their dependence on the shaft console values and external damping ratio.

Steady-state rotor oscillations with unbalanced disc under active gyroscopic torque

If we have a circular synchronous precession, it is appropriate to find solutions of motion equations in the polar coordinates

$$U = |A|e^{i(\Omega\bar{t} - \gamma)}, \quad \theta = |B|e^{i(\Omega\bar{t} - \delta)}, \quad (4)$$

where $|A|$ is an amplitude of disc linear displacement; γ is an angle of phase lag of this motions from the mass eccentricity vector; $|B|$ and δ are similar responses of disc angular motion.

If we insert (4) in (2) and (3), discard the time multiplying factor and input the actual parameters:

$$\begin{aligned}
 H &= \bar{I}_p - \bar{I}_T, \\
 a_1 &= (H\Omega^4 + 2c_3\Omega^2)\varepsilon - (3\chi c_2 H\Omega^3 \sin \beta)\tau + (3c_2 H\Omega^2 \cos \beta)\tau, \\
 a_2 &= (2\chi c_3\Omega^3)\varepsilon + (3\chi c_2 H\Omega^3 \cos \beta)\tau + (3c_2 H\Omega^2 \sin \beta)\tau,
 \end{aligned} \tag{5a}$$

$$\begin{aligned}
 b_1 &= (3c_2\Omega^2)\varepsilon + (-\Omega^2 + 6c_1)(H\Omega^2 \cos \beta)\tau - (6\chi c_1 H\Omega^3 \sin \beta)\tau, \\
 b_2 &= (3\chi c_2\Omega^3)\varepsilon + (-\Omega^2 + 6c_1)(H\Omega^2 \sin \beta)\tau + (6\chi c_1 H\Omega^3 \cos \beta)\tau,
 \end{aligned} \tag{5b}$$

$$\begin{aligned}
 \Delta_1 &= -H\Omega^4 + (6c_1 H - 2c_3)\Omega^2 + 12c_1 c_3 - 9c_2^2, \\
 \Delta_2 &= \chi(6c_1 H - 2c_3)\Omega^3 + 2\chi(12c_1 c_3 - 9c_2^2)\Omega,
 \end{aligned} \tag{5c}$$

we will get

$$|A| = \left[\frac{a_1^2 + a_2^2}{\Delta_1^2 + \Delta_2^2} \right]^{1/2}, \quad \gamma = \arctg \left[\frac{a_1 \Delta_2 - a_2 \Delta_1}{a_1 \Delta_1 + a_2 \Delta_2} \right], \tag{6}$$

$$|B| = \left[\frac{b_1^2 + b_2^2}{\Delta_1^2 + \Delta_2^2} \right]^{1/2}, \quad \delta = \arctg \left[\frac{b_1 \Delta_2 - b_2 \Delta_1}{b_1 \Delta_1 + b_2 \Delta_2} \right]. \tag{7}$$

We can get the natural frequencies of undamped system using the following equation

$$\Delta_1 = -H\Omega^4 + (6c_1 H - 2c_3)\Omega^2 + 12c_1 c_3 - 9c_2^2 = 0, \tag{8}$$

If we solve this equation, we can find critical speed for the direct precession

$$\Omega_k = \left\{ \left(3c_1 - \frac{c_3}{H} \right) \pm \left[\left(3c_1 - \frac{c_3}{H} \right)^2 + \left(\frac{12c_1 c_3 - 9c_2^2}{H} \right) \right]^{1/2} \right\}^{1/2}. \tag{9}$$

Formula (9) shows that critical speeds depend on the design parameters H, c_1, c_2, c_3 . Varying them depending on the requirements of technology processes, we can find optimal values of critical speeds required for efficient and safe operation of the rotary machine.

Amplitude-Frequency and Phase-Frequency Responses of Rotor Oscillations and Limiting Options of Rotor Geometry

Numerical computations of some typical rotor geometry options include the following dimensionless parameters: mass eccentricity and disc misalignment angle are equal to $\varepsilon=0.01$ and $\tau=0.02$, while the external damping ratio and console ratio are equal to $\chi=0.01$ and $a/L=0.25$. The findings of dependences $|A|(\Omega)$ and $\gamma(\Omega)$, $|B|(\Omega)$ and $\delta(\Omega)$ using formulas (6) and (7) show that the mass imbalance and disc misalignment have an impact on the rotor dynamics in the entire range of shaft speed. Thus, the amplitude-frequency and phase-frequency response curves are similar to the research

results of Benson (1983), but in case of thick disk ($H=-0.1$) the resonance amplitudes have limit values at the second critical speed, which are much less than at the first speed. The amplitude frequency responses of the linear rotor motion with thick disc are illustrated in Figure 2.

If we vary the values of design parameters $\varepsilon, \tau, \beta, H, c_1, c_2, c_3, \chi$ depending on $|A|(\Omega)$ and $|B|(\Omega)$, we can control the resonance amplitude and choose an appropriate operating speed range required for the process procedure. It is reasonable to derive an explicit formula for the residual phase angle of linear disc displacement at low speeds.

Figure 2: Linear motion amplitude. Thick disk at $H=-0,1$; $a/L=0,25$; $\varepsilon=0,01$; $\tau=0,02$; $\chi=0,01$

Source: Author

We can get (6) from (5a), (5b) and (5c) as follows,

$$\gamma_0 = \lim_{\Omega \rightarrow 0}(\gamma) = \arctg \left[\frac{-\frac{3}{2} \left(\frac{H\tau}{\varepsilon} \right) \sin \beta}{\frac{c_3}{c_2} + \frac{3}{2} \left(\frac{H\tau}{\varepsilon} \right) \cos \beta} \right] \quad (10)$$

The detailed formula analysis (10) demonstrates that the residual phase angle γ_0 depends only on three parameters: orientation angle of maximum misalignment line β , “imbalance ratio” $H\tau/\varepsilon$, characterizing a comparative role of disc misalignment and mass eccentricity, as well as design parameter ratios c_3/c_2 .

Figure 3 shows the dependence diagrams (10). You can see well that the residual phase angle can have any values in the range of $-180^\circ \leq \gamma_0 \leq 180^\circ$, especially if $\varepsilon \rightarrow 0$ (one, only disc misalignment).

Figure 3: Dependence of a residual phase corner of linear movement of a disk on the relation of imbalances

Source: Author

Figure 4: Dependence of the residual phase angle linear drive on the size of the cantilever

Source: Author

In the short range $|H\tau/\epsilon| < \frac{2}{3} \frac{c_3}{c_2}$, the zero values of residual phase angle γ_0 correspond to the values $\beta=0$ and 180° . Figure 4 shows the diagrams of residual phase angle γ_0 dependence on the console value a/L with constant orientation line angle values of the maximum disc misalignment β .

This means that as long as the console ratio a/L rises, the residual phase angle γ_0 decreases and

reaches a certain limiting value $\arctg \left[\frac{-\frac{3}{2} \left(\frac{H\tau}{\varepsilon} \right) \sin \beta}{1 + \frac{3}{2} \left(\frac{H\tau}{\varepsilon} \right) \cos \beta} \right]$, intrinsic to the one-bearing console rotor

case.

Thus, the processing rotor has a residual angle of linear translation phase displacement at low speeds, i.e. it is not necessarily in one and the same phase with the mass eccentricity line. At higher disc speeds, all curves of the thin disc linear displacement phase reach asymptotically the common limit value equal to 180° , and curves of angle displacement with asymptotes to phase motion angle are equal to $\delta \rightarrow -\beta$. These findings are extremely important from the practical point of view. E.g. on rotor balancing, the measuring results of linear displacement phase shift at low speeds are often used to determine the mass eccentricity line. Design engineers have to compare the phase measuring results both at low and high speeds to evaluate the level of disc misalignment effect depending on the level of phase difference from the value of 180° .

Conclusion

We have studied the steady-state oscillations of two-bearing console rotor with mass imbalance and disc misalignment. We have set up the differential equations of motion in view of external damping, which enabled us to describe correctly the amplitude-frequency and phase-frequency rotor responses, to evaluate the effect of combined generalized imbalances, relative dimension of shaft console area and external damping ratio on them. It has been found that the thick-disc rotor at highest critical speed has lower oscillation amplitudes than the rotor at lowest critical speed. Engineers may need the found expressions for the residual phase angle of linear disc displacement at low speeds to estimate experimentally the value and mass imbalance orientations on rotor balancing. The calculations of critical speed expressions and oscillations amplitude dependences on the speed enable us to choose an appropriate rotor operating speed range for the process procedure, as well as to control the resonance amplitudes. This rotor model is rather generalized as compared to the previous models and it can be easily applied to other rotor types.

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