ESTIMATING THE PARAMETER DELTA IN THE BLACK MODEL USING THE FINITE DIFFERENCE METHOD FOR FUTURES OPTIONS

Lucia Švábová[1]

Abstract: Financial derivatives are a widely used tool for investors to hedge against the risk caused by changes in asset prices in the financial markets. A usual type of hedging derivative is an asset option. In case of unexpected changes in asset prices, in the investment portfolio, the investor will exercise the option to eliminate losses resulting from these changes. Therefore, it is necessary to include the options in the investor´s portfolio in such a ratio that the losses caused by decreasing of assets prices will be covered by profits from those options. Futures option is a type of call or put option to buy or to sell an option contract at a designated strike price. The change in price of the underlying assets or underlying futures contract causes a change in the prices of options themselves. For investor exercising option as a tool for risk insurance, it is important to quantify these changes. The dependence of option price changes, on the underlying asset or futures option price changes, can be expressed by the parameter delta. The value of delta determines the composition of the portfolio to be risk-neutral. The parameter delta is calculated as a derivation of the option price with respect to the price of the underlying asset, if the option price formula exists. But for some types of more complex options, the analytical formula does not exist, so calculation of delta by derivation is not possible. However, it is possible to estimate the value of delta numerically using the principles of the numerical method called "Finite Difference Method." In the paper the parameter delta for a Futures call option calculated from the analytical formula and estimated from the Finite difference method are compared.

JEL Classification Numbers: G11, G13, G23, **DOI:** http://dx.doi.org/10.12955/cbup.v3.591

Keywords: Option, European call option, futures option, Finite Difference Method, Delta parameter, Delta hedging

Introduction

The main motivation for participants in the financial markets to avoid growth uncertainty is to look for new financial instruments. As the level of investment risk increases simultaneously, everyone operating in the financial markets has to react to market changes and to correct their investment strategy in time. Correctly used derivatives, e.g. options, can help investors increase their expected returns and minimize their exposure to risk. As written by Whaley (2006) and Hull (2012), the options were traded for the first time in 1973. Then, the volumes of their trade has risen rapidly all over the world.

Setting the right price for all types of options, from basic types of vanilla options to more complicated types of exotic options, is a very important part of derivative trading. As Hull (2012) wrote, the Black-Scholes-Merton model for option pricing is at present one of the most famous tools for valuation of derivative contracts. Since its time of introduction, the volume of option trades has significantly increased. Standard types of options are traded actively, and new types of options bring other possibilities for investors to hedge their investment portfolios. On the other hand, some types of exotic options could be very complicated and difficult to price, so that the Black-Scholes model in his basis could not be used to set their price. For some of these options, the analytical pricing formula does not exist. Due to this fact, it is much more complicated for the investors to create and correct the investment strategy if the values of the parameters used for investment strategy revision cannot be computed analytically. Thus, the numerical methods come into play. Using these methods, the prices of exotic options can be estimated, and, therefore, the values of the required parameters can be obtained. One alternative of numerical options pricing is a finite difference method based on the Black-Scholes differential equation. Using the principles of the finite difference method, it is possible to estimate the values of the aforementioned parameters numerically, as mentioned by Ďurica & Švábová (2014b).

[1] RNDr. Lucia Švábová, PhD. University of Žilina, Faculty of Operation and Economics of Transport and Communications, Department of Quantitative Methods and Economic Informatics, Univerzitná 1, 010 26 Žilina, Slovak republic, email: lucia.svabova@fpedas.uniza.sk

Types of options

As defined by Whaley (2006), an *option* is an instrument giving its owner the right, but not the obligation, to buy or sell something at an advance fixed price. A *call/put* option gives the holder the right to buy/sell the underlying asset at the strike price before the expiration time. The price agreed by the seller and the buyer in this contract is known as the *strike price*, and the date, when the right given in the option will expire, is known as the *expiration date* or *maturity*. It is important to emphasize that the option gives his holder the right to do something, and the holder is not obligated to exercise this right. As said by Jarrow & Rudd (1983), the ownership of the option provides only the right to do something not the obligation, and this is why the owner of the option has to pay something for the conclusion of such contract. An option buyer pays the option premium for the right to buy or sell the underlying asset.

Hull (2012) wrote that all the simple derivatives, such as American or European options, are together called *plain vanilla options*. But, besides these simple types of options, there exists a large number of nonstandard products, which have been created in the over-the-counter derivatives market. One other type of option contract is the *Futures option*. In this option, the underlying asset is the Futures contract; that means when a Futures option is exercised, a Futures position is opened at the predetermined strike price in both the buyer and the seller’s account. Depending on whether a call or a put is exercised, the option buyer and seller will assume either a long position or a short position.

Basic principles of option pricing

The most popular pricing model is the Black-Scholes model introduced for the first time by Black & Scholes (1973). The Black-Scholes model was improved by R. Merton in 1973. As Hull (2012) wrote, his approach was different from that of Black and Scholes’. It was more general because he did not rely on some assumptions for the underlying asset. Merton considered, in his model, the assumption of the dividend payment possibility from the underlying share of the derivative. For Futures options’ pricing, the Black model is used. It is based on the fact that for Futures option, we replace, in the Black-Scholes-Merton (BSM) model, the dividend yield with the risk-free interest rate and the underlying share price with the underlying Futures price.

As said described by Švábová (2013), another way to price the options is to use the numerical methods. There exist several types of numerical methods, which utilize the price from the majority of the options including the more complex exotic options that are “path dependent” and can be correctly estimated. One alternative method for numerical options pricing is a Finite difference method. This method is based on Black-Scholes-Merton partial differential equation and represents one possibility of option pricing. As Hull & White (1990) wrote, this numerical method could be very useful for the option pricing especially for more complex types of exotic options whose price have to be set numerically.

Black-Scholes-Merton model

It is not easy to determine the right price of an option in practice. A big number of pricing models were generated to solve this problem. As Whaley (2006) and Hull (2012) wrote, the Black-Scholes-Merton model is used to calculate a theoretical price of the option using the five key factors of an option’s price—underlying stock price, strike price, volatility, time to expiration, and risk-free interest rate. The Black-Scholes-Merton model is derived under the validity of some assumptions.

Under the assumptions mentioned by Hull (2012), the Black-Scholes-Merton differential equation is valid for a European type of option with an underlying asset paying continuous dividend yield:

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial S}(r-q)S+\frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2=rf. \qquad (1)$$

In equation (1), the Merton's approach is used; it allows the existence of continuous dividend yield *q*, paid from the underlying asset. In equation (1), the following notations are used:
f is the price of the derivative (option);
t is the time;
S is the price of the underlying share;
r is risk-free interest rate;
q is dividend yield rate;
σ is the stock price volatility.

This equation has infinite number of solutions based on the selected initial conditions. As Hull (2012) wrote, for a European call option, the solution of the BSM equation is a formula:

$$c=Se^{-qT}N(d_1)-Xe^{-rT}N(d_2); \qquad (2)$$

where $\mathrm{N}(\mathrm{d}_1)$ and $\mathrm{N}(\mathrm{d}_2)$ are the values of the distribution function of a standard normally distributed random variable in points d_1 and d_2 given by Hull (2012).

The nomenclature used is valid as before, for (1) and (2). Moreover, *X* is the strike price, and *T* is the time of maturity of the option. Equation (2) is the most famous solution of the BSM differential equation (1), and is known as the Black-Scholes-Merton model (pricing formulas) for the price of European call options.

Black model

By replacing the dividend yield *q* with the risk-free interest rate *r* and the underlying share price *S* with the Futures contract value *F*, in (1) and (2), we obtain the Black model for Futures option pricing as follows:

$$\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2=rf. \qquad (3)$$

The solution of equation (3) is the analytical formula for Futures option pricing:

$$c_f=Se^{-rT}N(d_1)-Xe^{-rT}N(d_2); \qquad (4)$$

where $\mathrm{d}_{1,2}$ are given by Hull (2012).

Delta-hedging

The option traders use sophisticated hedging schemes for hedging their portfolio. As the first step, they attempt to make their portfolio immune to small changes in the underlying asset price in the next small time interval. As Hull (2012) wrote, this strategy is known as *delta-hedging*. Then, they look at the rate of change of delta with respect to the price of the asset; this rate is known as the parameter *gamma*. By keeping gamma close to zero, a portfolio can be made relatively insensitive to fairly large changes in the asset price (Hull, 2012).

Parameter *delta* of the derivative is defined as the rate of change of its price with respect to the underlying asset price. As written by Hull (2012), it is important to realize that the position of investor remains delta-neutral for only a relatively short period of time. This is because delta changes with both changes in the Futures price and the passage of time. In practice, when delta hedging is implemented, the hedge has to be adjusted periodically. This is known as *rebalancing*. Hedging schemes such as this that involve frequent adjustment are known as *dynamic hedging schemes*.

The delta of futures call option

As written by Ďurica & Švábová (2012b), the delta of Futures option is the rate of change in the value of this Futures option with respect to the change in the underlying Futures. This change can be calculated using the partial derivative of the option price with respect to variable F. Therefore, the delta of a call Futures option is:

$$\Delta = \frac{\partial \mathrm{c_f}}{\partial \mathrm{F}} = \mathrm{e}^{-\mathrm{rT}} \mathrm{N}(\mathrm{d_1}) \tag{5}$$

The delta for a call Futures option has a positive value and is an increasing function of the underlying asset price. The behavior of parameter delta in European call option, for various times to maturity and also for some types of exotic options, has been studied by Ďurica & Švábová (2012; 2013; 2014a).

Finite difference method

The Black-Scholes-Merton equation (1) is a partial differential equation valid throughout the whole given domain. As written by Švábová (2011), the aim of the finite difference method is to reduce the domain of option values definition into a finite number of points and to replace the partial derivations in equation (1) with given differences. More specifically, the explicit finite difference method uses such approximations of partial derivations which, by being substituted to equation (1), lead to the formula for calculating the price of the derivative directly at that point:

$$f_{i,j} = \frac{2\Delta S^2 f_{i+1,j} + \Delta t \Delta S (r-q) S_j \left(f_{i+1,j+1} - f_{i+1,j-1}\right) + \sigma^2 S_j^2 \Delta t \left(f_{i+1,j+1} + f_{i+1,j-1} - 2 f_{i+1,j}\right.}{2r\Delta t \Delta S^2 + 2\Delta S^2} \tag{6}$$

Equation (6) gives the relationship between three values of the option at time $i\Delta t$ in points $f_{i,j-1}$, $f_{i,j}$, and $f_{i,j+1}$; and, one value of the option at the following time at point $f_{i+1,j}$.

In the grid of option values used in the explicit finite difference method, we move from the end of the grid to the beginning. At each node, we calculate the value of the derivative using equation (6). In this explicit approach, the values in all the interior points in the grid are given by the values of the neighboring points in next time step in the grid. The situation is described by Ďurica & Švábová (2014b; 2012a). Using this approach, we repeat the calculations until we get the values on the left side of the grid at time zero.

Using the Explicit finite difference method for calculating Futures options' delta

As mentioned earlier, the value of delta can be calculated by a partial derivation of the option price with respect to the price of the underlying Futures, or we could calculate delta using the differences in the grid. Then, delta for every interior point of the grid will be the difference between the values of two neighboring points, upper and lower, divided by two lengths of the share price subinterval:

$$\Delta = \frac{\partial \mathrm{c_f}}{\partial S} \approx \frac{c_{f(i,j+1)} - c_{f(i,j-1)}}{2\Delta S} \tag{7}$$

This is very useful, especially, in such cases of more complex exotic options, in which the option price have to be set only by numerical method, because the analytical formula does not exist. This is a very frequent situation because some options are so complex that the basic Black-Scholes-Merton model cannot be used for their pricing. So, they have to be priced only numerically. Then, the value of parameter delta should be calculated from the numerical estimates of the option prices too.

Results

We will present an estimation of the value of the parameter delta for Futures option using the differences in the finite difference method grid as an example. Let us assume a Futures option with the strike price of 50. This option will expire in one year. The risk-free interest rate and the volatility are both 10% per year.

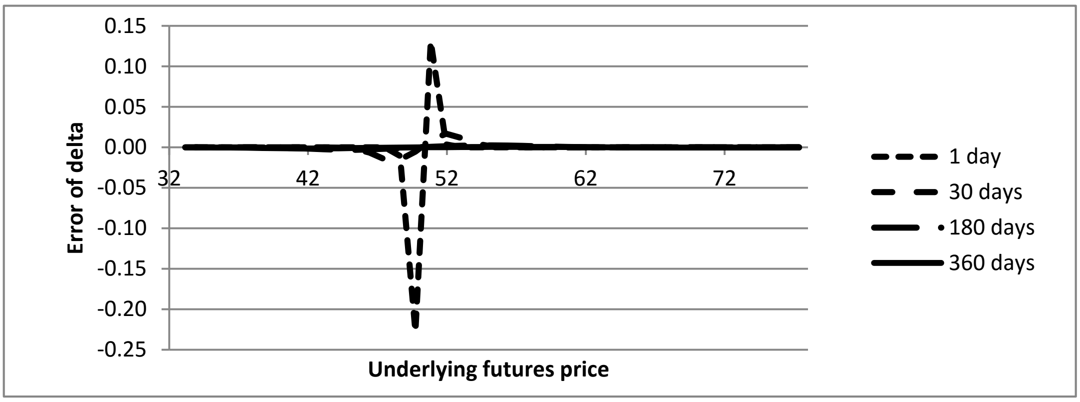
We estimated the price of this option by the explicit finite difference method with a grid where the 1-year life time of the Futures option was divided into 360 subintervals with the length of 1 day, and the share price interval from 0 to the maximum of 50 was divided into 200 subintervals with a step of 0,25. Using the approach mentioned above, we obtained all the option prices in the grid points. Then, we used these values to estimate the parameter delta from the option prices using equation (4).

In Figure 1, the errors of delta values are shown. These are the differences between the delta values calculated from the pricing formula as the partial derivation (7) and the values estimated numerically using the Finite difference method grid using (6).

As we can see, the differences between these two values of delta are very small, close to zero. The biggest differences are for 1 day to maturity and close to the strike price of an Futures option, where delta is very sensitive, and its values are close to 0 or 1.

This is a very good result, which means that the numerical method can be used successfully for estimating the values of parameter delta for Futures. Therefore, this parameter is the most widely used hedging parameter. So, for more complex types of exotic options, this numerical method can bring a possibility of estimating the value of the hedging parameter numerically.

Figure 1: Error of parameter delta values



Source: Author

Conclusion

As a result of our research, we can summarize that the finite difference method can be used successfully for estimating the values of the parameter delta used for hedging. This is valid not only for the parameter delta, but also for parameter gamma or theta because we can use the differences instead of the partial derivations required in the Black-Scholes-Merton model. As we proposed, using more division points in a grid generates more accurate results. This approach should also be used for more complex options without the analytical formula, so that we could estimate their delta values numerically from the explicit finite difference method.

Acknowledgements

This article was written in response to the project co-funded by the EU called "The quality of education and human resources development as pillars of a knowledge society at the Faculty PEDAS, ITMS 26110230083."

References

Black, F. & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, 81*(3), 637-654.

Ďurica, M., & Švábová, L. (2012a). Using the Finite difference method for Futures option pricing. *International Scientific Conference IMEA 2012 Full Papers Proceedings*, 214-219.

Ďurica, M., & Švábová, L. (2012b). Parameters delta a gamma of Black model of Futures option pricing. *International Scientific Conference MMFR 2012 Full Papers Proceedings*, 125-132.

Ďurica, M., & Švábová, L. (2013). An improvement of the delta-hedging of the Futures Options. *International Scientific Conference FMFFI 2013 Full paper proceedings*, 140-148.

Ďurica, M., & Švábová, L. (2014a). Delta and Gamma for Chooser Options. *International Scientific Conference Applications of Mathematics and Statistics in Economics AMSE 2015 Full paper proceedings*, 75-84. doi:10.15611/amse.2014.17.08

Ďurica, M., & Švábová, L. (2014b). Estimating of the parameter delta of the Black – Scholes model using the Finite difference method. *International Scientific Conference MSphere 2014 Full Papers Proceedings.*

Hull, J. C., & White, A. (1990). Valuing Derivative Securities Using the Explicit Finite Difference Method. *Journal of Financial and Quantitative Analysis*, 25, 87-100.

Hull, J. C. (Ed.). (2012). *Options, Futures and Other Derivatives (8th ed.).* England: Pearson Education press.

Jarrow, R. A., & Rudd, A. (1983). *Option pricing*. USA (Ed.): Richard D. Irwin, INC.

Merton, R. C. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science*, *4*(1), 141-183.

Švábová, L. (2011). Using of the Finite Difference Method for European Call Option Pricing. *International Scientific Conference Actuarial Science in theory and in practice 2011 Full Papers Proceeding*, 115-122.

Švábová, L. (2013). Pricing of selected options using the finite difference method. *International Scientific Conference Actuarial science in theory and in practice 2013 Full Papers Proceeding*, 152-160.

Whaley, R. E. (2006). *Derivatives: Markets, Valuation, and Risk Management*. New Yersey (Ed.): John Wiley & Sons.