3D INNOVATION TECHNOLOGIES IN EDUCATION
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Abstract:
Introduction: Studying the discipline of stereometry is difficult for students who do not have spatial imagination. To overcome this problem, software applications are being created in addition to traditional training methods to improve the understanding of 3D geometry.

Objectives: The subject of this article is the presentation of the functionality and tools of a created software application in teaching stereometry, with the help of which students will construct, observe, measure and study 3D geometric shapes, as well as create dynamic images.

Methods: The methods: stereoscopy, polygon triangulation, cleaning/extrusion, and level of detail were used to create 3D geometric shapes via the software application designed for the secondary education of the Bulgarian students in stereometry

Results: The software application created the following objects in 3D space: regular prism (three to six sides), general three-sided prism, special four-sided prism, regular pyramid (three to six sides), cube, sphere, cone and cylinder. The parameters of any geometric object can be adjusted, including the height and length of the sides, the slope of the sides, the number of sides, etc. Each object can be gradually opened or closed from its main network to the geometric object. Each parameter correction is interactive and visible immediately. Geometric shapes can be rotated in space, zoomed in to or out of, and viewed as a solid object or transparent net, as well as have the ability to change colors.

Conclusion: With the software application created, new opportunities for teaching and learning in the subject of stereometry are provided, which helps students to more easily understand spatial geometry and the ability of teachers to explain abstract problems in 3D geometry.

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Introduction
Modern information technologies allow different visual information to be displayed in three-dimensional space (Piovesan et al., 2012). Information about the surrounding world is perceived by sight. In order to achieve maximum accuracy in the perception of information, it is necessary to provide a visual perception that is as close as possible to reality. When a 2D image is observed, the brain attempts to reproduce a 3D situation where information is lost, as each person subjectively interprets what they see according to their experience, knowledge, and other factors (Nedeva et al., 2017). 3D image modeling is a key element in today’s research (Beluce et al., 2015). The use of 3D modeling and animation, being actively involved in education provides teachers with new tools to help learners more easily perceive the learning material, increase their motivation, and help accelerate the learning of large volumes of knowledge (Petkov, 2017). Among the benefits of using 3D technology in education can be noted the following (Martin-Gutiérrez et al., 2017):

- Teachers teach with the help of high-tech educational tools, thus saving time for explaining complex concepts and processes.
- The visualization of the "complicated" themes in the curriculum helps students to better understand the material being studied.
- Including 3D models of processes and objects in traditional ways of learning introduces innovation into the "routine" learning process, enhancing motivation for learning (Getso et al, 2017).
- It draws students' attention to lessons, increases concentration and attention, improves and accelerates the perception of the material studied.
- Facilitates systematization of knowledge.
- It facilitates the learning of more information, which has a positive effect on test results and exams.

The purpose of this article is the presentation the basic functions and tools of a new software application for teaching students in stereometry in the study of 3D geometric shapes and to make the training more efficient and effective.

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Literature review

The methodology for training school students on stereometry includes the following two approaches: constructivism and connectivism (Bada, 2015). Constructivism is an approach where the learner uses previous knowledge to build new ones. School students can be divided into groups and given specific tasks in order to learn to work together in a team. In this type of training, the teacher is the most important; he is an organizer and supervises the learning process. The inverse approach of constructivism is the methodology of connectivity, which does not require knowledge acquired in the past (Siemens, 2005). The approaches used in it are mainly: monitoring and thinking. Using 3D technology in the middle course allows students to: explore, observe, experiment, discover, and suggest (Tővissy et al., 2016). They enable the learner to work independently and in a team while being supervised by the teacher. In some cases, it may be that the teacher becomes a mere observer. He does not explain before the students themselves try to make a concrete solution to the problem. Using the right software, the teacher will not waste much time preparing lessons. In the traditional way of teaching a sheet of paper and a board, students are struggling to study stereometry because they do not see the structure of the figures well. For example, when studying a cube, the student does not see the base well, namely that it is a square, he/she can accept the base for a parallelogram. Some of the most popular mathematical software are GeoGebra (GeoGebra, 2018), DALEST (Boychev et al., 2007) and CABRI 3D (Subroto et al., 2011).

Until now, the research approach in Bulgarian education is specifically related to the mathematical dynamic software Geogebra. Only initial computer literacy is sufficient to use it (GeoGebra, 2018). The classic method, i. e. the use of chalk, line and pergel ensure the understanding of the drawing in stereometry. The construction of the drawing takes place in the imagination but in a flat structure. With modern computer technologies for construction, new possibilities are found for drawing geometric figures. CABRI 3D allows for the building of geometric figures in the virtual space (Subroto et al., 2011). Thanks to it, a new approach is emerging, such as design and creation (Rososczuk, 2015). It is possible for a new spatial creation of the figure. Three-dimensional manipulations with geometric shapes are performed using this program. After creating the geometry object, the program allows interactivity, and the drawing can be brought back one step back or forward. It allows the object to be viewed at different angles.

The work presented in the DALEST project is the result of the joint work of several European universities (Christou et al., 2007). The aim of the project is to develop dynamic 3D software for studying the stereometry in secondary school. The software has a convenient interface that helps the student interact with the objects. DALEST allows objects to be drawn using solid/wireframe charts. The constructions in the program are made from: dots, lines, surfaces and other primitives. The aim is to help the student explore and discover the geometric properties of the figures. The software consists of several applications, such as: Cubix, slider, Stuffed toys, Cubix editor, Cubix shadow, etc.

Methods for modeling three-dimensional objects

The main goal of 3D technology is the realization of real models via a computer. There are three types of objects: light sources, objects and cameras. The triangle is the simplest polygon, as it is known from mathematics; three vertices of space lie in a plane that can form a triangle. This primitive is used to implement more sophisticated objects. There is an opportunity to implement 3D technologies in the field of mathematics. For the realization of this kind of technology it is necessary to use the most suitable techniques in three dimensional programming, such as: the triangulation of polygons level of details, sweep model, and others. Most of them are applied on surfaces rather than on multiple points.

Sweep/Extrude model - In this method, a three-dimensional object is created by a plane section and a rule defining how the cross section moves in space to create the body (Gindis, 2016).

Triangulation of polygons - With this method, surfaces can be created with a large number of triangles. It applies to the already created polygons (Klawonn, 2012). In other words, each created object can be divided into a large number of triangles.

Level of Detail (LOD) - This method stores an object with a different number of triangles, and then decides which resolution will be applied to the object (Gordon et al., 2019). The LOD approach can be applied to the representation of each object, such as: polygons, splines, voxels, implicit surface, etc. However, it is mainly applied to polygons. At this stage polygons are most commonly used in
computer graphics. There are many algorithms (Luebke et al., 2003) which are mostly applied to polygons, to be divided into a large number of triangles. These methods are used to create geometric bodies such as: a pyramid, a cube, a sphere, a cylinder, a prism, and so on. Stereometry objects can be represented as solid and wireframe. Many 3D graphics software uses wireframe to represent objects. This type of graphics consists of a number of peaks and angles connecting these peaks. On the other hand, solid objects are represented with a larger density. There are algorithms that can be used to convert from wireframe to solid. The process is called solidifying wireframes (Srinivasan et al., 2005). The basic solids models are: Primitive Instancing; Sweep presentation; Boundary representation (b-reps); Constructive Solid Geometry (CSG). In the first model, the modeling system is defined by a series of primitives with a solid shape. In the b-reps model, the object is described in terms of its boundaries: as peaks, edges, and lines. The CSG model has a combination of primitives that can be used to create complex 3D objects. Solid modeling uses topological information, in addition to the geometric, to represent the objects completely. The advantage of solid models over others is that the solid model leads to a more accurate design. The downside is that there is information that cannot be seen.

Results
Basic functions of the created software application for teaching stereometry
A new software application has been developed to educate school students in the field of stereometry, incorporating some of the benefits mentioned in the software products presented above, as well as new functionalities. The application uses the JAVA object-oriented language for software implementation. This software implements the material included in the stereometry textbooks of the Archimedes Publishing House for secondary education of the Bulgarian students. The material studied is divided into modules of varying difficulty, as well as the opportunity to test students' knowledge.

The main functionalities of the created software application are the following:

- Stereoscopic visualization of the geometric shapes (prism, cube, pyramid, cone, sphere, geometric fractals, etc.) in order to study their basic elements such as: top, edge, wall, section, radius, etc.
- Ability to represent geometric shapes from solid in wireframe.
- Possibility of selection and manipulation. Once the object is selected, its position, orientation, size, density and color are determined.
- Parametric representation of the selected object by setting values of certain parameters, such as: height, length, width, etc.
- Possibility to unfolding the studied object in order to better understand it.
- Ability to work in the "cross section" mode of the selected figure. The student selects the vertices of the figure through which to build the cross section and the application creates it.
- Performing calculations such as: finding the surface area and volume of the geometric object studied, as well as converting the units of measurement.
- Allows students to work in two modes: normal and dissection.
- Ability to switch from 2D to 3D.
- Possibility for the student to “enter” into the studied object and observe it from within.
- An error message is displayed when incorrect input data related to the studied object is entered.
- A library of geometric shapes has been created, such as: prism, cube, pyramid, cone, sphere, geometric fractals, etc., which is open for future expansion.
- The application has a user-friendly interface implemented through: menus, buttons, tables, sliders, etc.
- Help menus have been created to help students use the software.

The advantages of the developed software application are the following:

- Development of spatial thinking in students, where the object of study can be viewed at different angles, be reduced, enlarged;
- Intuitive perception of the studied figures;
- Holding students' attention to the object being studied;
- Possibility for creative expression of students;
- Ability to test students' knowledge.
Examples for creating 3D geometric shapes

The software application contains a number of tools that can be used to construct 3D geometric shapes, as well as to manipulate its parameters. The article shows examples of three types of geometric shapes: pyramid, prism and cube.

Figure 1 shows a pyramid in both application modes: normal and dissection. The pyramid (Figure 1a) is tilted and each side of the pyramid is colored differently. The software is designed such that the drawing style of any object can be in a ‘solid colour’ view (Figure 1a) or in a ‘transparentline’ view (Figure 1b), and students can select and color the faces of the figures.

Figure 1: Solid view and dissection mode of a pyramid

![Figure 1](image)

Source: Author

In Figure 2a a cube is shown inscribed in a regular quadrilateral pyramid, and in Figure 2b – parallelepiped. The lower base of the figure inscribed lies on the base of the pyramid, and its upper base is a cross-section parallel to the base of the pyramid. The application enables students to track how the body inscribed in the pyramid changes from a cube to a parallelepiped and vice versa by introducing the length of the edge of the prism. After previewing, the application automatically displays the length of the edge of the figure inscribed.

Figure 2: Cube and parallelepiped inscribed in a regular quadrilateral pyramid

![Figure 2](image)

Source: Author

Figure 3 is showed a proper prism. To visualization the proper prism, the student introduces: the base of the prism, which is a regular polygon and its height. When entering the height, the prism can be presented in two ways: straight (Figure 3a) and tilted (Figure 3b).

The software is designed so that students can rotate a geometric object with respect to the three axes and thus gain a complete view of the object being studied. The software also allows the elements of the object to be dynamically modified for the student to explore its properties. The folding and unfolding process is an exciting area of stereometry, but it is difficult for students. With the help of the proposed software, this problem is solved.
Figure 4 shows the process of unfolding a cube. Through unfolding a cube, students will be able to see the base of the cube well as a square and not as a parallelogram. At the same time, they will understand the structure of the figure and will be able to try to derive the formulas for the surface of a cube and its volume. The application enables them to show two types of cube templates, namely from cube to figure, which can lie on a square grid and vice versa.
Conclusion
The created software application in teaching stereometry using the 3D technology enables students to observe, investigate and to discover for themselves the dependencies of the studied geometric objects. This makes the learning process more interesting, motivating, has increased quality and effectiveness. The software application is intended for both students and teachers. The application allows the student to work independently and in a team while being supervised by the teacher. The new approach to the spatial study of geometric shapes using the 3D technology complements the traditional teaching style and it is expected to lead to better results in the study of stereometry. The application created is open to future development, expansion and modification to meet all new requirements from teachers and students.

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