PRICE–VOLUME DEPENDENCE OF BITCOIN AND ITS FRACTAL ANALYSIS

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Abstract: Nowadays Bitcoin as cryptocurrency takes a significant place on the global financial markets. This paper analyzes the Bitcoin closing prices and traded volume during the period from December 28, 2013 to January 22, 2019. This period is known as a period with rapid increasing of the Bitcoin closing prices, mainly in the second half of the year 2017. The aim of this paper is twofold. First, we compute the Hurst coefficient to discover the close price dynamics and traded volume using a fractal point of view. We have discovered an anti-persistent behavior in the traded volume and random character of bitcoin closing prices. Second, we propose an analysis of the relationship between the close prices and traded volume. Our findings show how changes in the high-price period differ from changes in the low-price period. We also found that high prices caused investors to be afraid to trade due to possible rapid decrease in bitcoin closing prices.

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Introduction

Bitcoin is a very popular virtual currency (cryptocurrency) that is managed by an open-source software algorithm that uses the global internet network to create Bitcoin and to record and verify its transactions. The popularity of Bitcoin can be attributed to its novel features, transparency, simplicity and low cost of foreign exchange, speed of conversion, anonymity, absence of third parties, and decentralization. Simple manipulation and management of cryptocurrency are other positive features of the modern phenomenon. Principles of cryptography are used to control the creation and exchange of Bitcoin. Users of Bitcoin can store it in local wallets on a PC or on a smartphone using open-source software or in an online wallet. Nowadays Bitcoin can be used to buy goods or services worldwide. We can note that the number of companies that accept Bitcoin as payment is growing. Bitcoin is very popular between academics that would like to predict Bitcoin behavior and that analyze it in the statistical or behavioral sense.

The aim of this paper is to examine the impact of the conflict between randomness and determinism in the cryptocurrency market. The paper is organized as follows. The next section presents the literature review. The methodology and data sections describe the data and methods used. The results section discusses our findings and the final section concludes the paper.

Literature review

Financial markets offer many financial instruments to invest in. Bitcoin can combine the advantages of both commodities and currencies on the financial markets as is shown in the paper by Dyhrberg (2016). Therefore, Bitcoin can be a useful tool for portfolio investors or risk managers. Moreover, the cryptocurrency market offers many opportunities to analyze it mainly for academicians. The main problem is how to predict future Bitcoin prices. Panagiotidis *et al.*, (2018) used a Lasso regression approach to analyze twenty-one factors that can have an influence on Bitcoin returns. The authors discovered that uncertainty in the European economic policy, the NIKKEI index and a negative google trend were the main determinants for Bitcoin returns during the analyzed period. Persistence, predictability or positive correlation between its past and future values on the cryptocurrency market was analyzed in the paper by Caporale *et al.* (2018). The authors discovered the predictability of Bitcoin markets and therefore they concluded that the cryptocurrency market is inefficient during the analyzed period July 17, 2010 to May 2, 2018. Urquhart (2017) showed that the price and volume of Bitcoin have a significant positive relationship with price clustering. El Alaoui *et al.*, (2019) used an unconventional method to analyze and examine the dependence between Bitcoin price changes and volume changes. They used a multifractal de-trended cross-correlation analysis. The authors discovered that the level of correlations is high and therefore the Bitcoin volume can predict the dynamics of the Bitcoin price change. But it is still unclear whether Bitcoin prices and traded volume have similar fractal properties and if the influence of bear markets or bullish markets has a similar impact on bitcoin price return or on the traded volume.

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Our results show that Bitcoin price and trading volume have different fractal characteristics and that there exists an asymmetric dependence between bitcoin price changes and changes of the traded volume.

Data and Methodology

We have employed the daily close price and traded volume data for Bitcoin against the US dollar (BTC/USD). The data spans a period from December 28, 2013 to January 22, 2019. The data was obtained from the data provider CryptoDataDownload (2019). Figure 1 shows closing prices and volume historical data for the whole analyzed period. As we can see in Figure 1, Bitcoin closing price decline has long been below USD 5,000 until the break in 2017, when it reached record values and approached USD 20,000. Year 2017 was the year when the wider public began to be interested in the cryptocurrency, other than computer scientists and investors. After a significant increase in the value of Bitcoin in December 2017, the value of Bitcoin fell mainly during January and February 2018. Later, we can see the ups and downs of the BTC price, but Bitcoin value has been steadily declining since December 2017. Bitcoin reached the lowest value on January 15, 2015, namely 120 USD. On the contrary, it has achieved the highest value during the analyzed period on December 16, 2017 when Bitcoin reached its historical maximum value of 19,650 USD.

Figure 1: Bitcoin Close Price and Traded Volume.
Data period: 12/28/2013 to 01/22/2019

Source: Author's calculation

In the second half of the year 2017, the price of BTC/USD started to rise sharply and therefore we have decided to divide the analyzed period into four segments. All-time period start on December, 27, 2013. The first period finishes on August 7, 2013 when the closing price exceeded a value of 3300 BTC/USD. The second period finishes on October 18, 2017 when the closing prices were around 5000 BTC/USD. During December 2018, Bitcoin's closing prices achieved approximately a four-fold increase compared to its August prices. The whole analyzed period finishes on January 17, 2019. We have 1322, 1393, 1442 and 1849 observations to analyze BTC closing prices and volume behavior.

During these periods it is not clear which fundamental process is the driver of this system. Sometimes we can observe certain aspects in real processes that give us evidence about the existence of an underlying more complex process. A problem arises when a given time series is related to an underlying more substantial process. We need to determine whether the underlying process is driven by a deterministic set of equations or by a stochastic system, or whether the process is self-similar. A question is if it is the difference between a deterministic and stochastic process and whether it is possible to make this distinction based on empirical observations.

Specific definitions of the deterministic and stochastic stationary time series were introduced by Cutler (1997). The Self-similar process was defined by Harte (2001) and it was characterized by the Hurst coefficient H ($0 < H < 1$) that enables us to distinguish the type of process. When H is greater than $\frac{1}{2}$, the process shows long range dependence. When H equals to $\frac{1}{2}$, the process is white noise. If H is less than $\frac{1}{2}$, the process exhibits a short range dependence with negative autocorrelations. The impact of the present on the future is expressed as the equation:

$$C(t) = 2^{(2H-1)} - 1, \quad (1)$$

where C is the correlation of all one-day returns with all future and past one-day returns or correlation of changes in time t with all increments of time t that precede and follow it (Peters, 1994; Peters, 1996).

We used a R/S analysis to detect the long-range dependence and to compute the Hurst coefficient H and its expected value $E(H)$ as described in the papers by Peters (1996) or Robinson (2003). We verified the null hypothesis: The time series is a random walk. If the Hurst coefficient H and its expected value $E(H)$ is approximately equal, it means the time series is independent and random during the analysed period (Hurst exponent is insignificant). If the Hurst coefficient H is greater (smaller) than its expected value $E(H)$, the time series is persistent (antipersistent) (Hurst coefficient is significant). If the series exhibits a persistent character, then the time series has a long memory and the ratios R/S_n will be increasing. If the ratios R/S_n are decreasing, the time series will be antipersistent. The “breaks” may signalize a periodic or nonperiodic component in the time series with some finite frequency. We calculated the V -statistics for precisely estimating where this break occurs (Peters, 1996).

$$V_n = (R/S)_n / \sqrt{n} . \quad (2)$$

The R/S analysis will be used to detect long range dependence of Bitcoin closing prices and traded volume. The dependence between changes of Bitcoin closing prices and changes of traded volumes was analyzed using quantile regression (Alexander, 2008). If the data does not follow the normal distribution, a quantile regression approach is appropriate to analyze the dependence structure of the selected variables. We decided to analyze the impact of changes in the closing prices of Bitcoin on changes in traded volume over the three periods described above. We have realized two regression models for each period, one before and one after the break date. Our quantile regression model is

$$\Delta \text{Volume}(q) = \alpha(q) + \beta(q) \Delta \text{ClosePrice} + \varepsilon, \quad (3)$$

We have evaluated the quantile regression with intercept α across seven quantiles $q = \{0.05, 0.1, 0.25, 0.5, 0.75, 0.90, 0.95\}$.

The model (3) allows us to test how the Bitcoin market reacted during bearish/bullish or normal markets conditions. The lower/upper quantiles represent bearish/bullish market conditions. Normal market conditions are represented by the median.

Results

We start by examining the R/S analysis results of the daily closing prices of Bitcoin expressed in USD and daily trading volume. During the period 12/27/2013 – 08/07/2017 the Hurst coefficient H was estimated with a value of 0.554. The expected value of the Hurst coefficient was $E(H)=0.585$ and the standard deviation of $E(H)$ was 0.028. The Hurst coefficient for the daily closing price data is -1.114 standard deviations away from its expected value. This result is non-significant at the 95% significance level. We can say that the estimated Hurst coefficient approaches its asymptotic limit value 0.50. Similarly, the Hurst coefficient was non-significant during each analyzed sub-period, see Table 1. The absolute value of the t -statistics is always less than 1.96. This suggests that the Bitcoin closing prices had a random behavior. When we look at correlation C between increments (Table 1), the lowest value 0.053 was estimated during the period 12/27/2013 – 10/18/2017. The highest value equal to 0.118 was recorded for the whole analyzed period. However, correlation between increments for the whole analyzed period was close to zero. It means there is no long-memory effect in daily values of Bitcoin closing prices. Finally, changes in bitcoin closing prices follow a non-stationary process.

The R/S analysis of Bitcoin volume values shows the Hurst coefficient equal to 0.273 with an expected value 0.585 for the first analyzed period (see Table 2). The standard deviation is equal to 0.028. The Hurst coefficient is 11.338 standard deviation bellow its expected value and it is strongly significant at the 95% significance level. We have obtained similar results for each analyzed period. All Hurst coefficients are significant, the values are from the interval $0 < H < 0.5$ and therefore the volume data exhibits an anti-persistent character. A correlation between increments C in volume time series takes negative values. This behavior points to relaxation processes (Peters, 1994; Peters, 1996). The relaxation process is obviously characterized by two trends. One trend is based on sentiment and the other on value. This means that the market as a whole increases its profits and the improved sentiment increases Bitcoin's volume to the new real value. These results tell us that the Bitcoin market as a whole may have many different parallel relaxation times in response to the same information. In addition, each investment horizon has the same structure of volatility.

Table 1: Hurst coefficient for BTC/USD Close Prices

<i>BTC/USD Close Prices</i>	12/27/2013 - 08/07/2017	12/27/2013 - 10/18/2017	12/27/2013 - 12/05/2017	12/27/2013 - 01/17/2019
<i>H</i>	0.554	0.538	0.569	0.581
<i>E(H)</i>	0.585	0.574	0.581	0.575
<i>t-stat</i>	-1.114	-1.349	-0.46	0.248
<i>Standard Deviation</i>	0.028	0.027	0.026	0.023
<i>C</i>	0.078	0.053	0.1	0.118
<i>Rsquare</i>	0.994	0.988	0.993	0.992
<i>Rsquare E(H)</i>	0.997	0.998	0.998	0.998
<i>Number of observations</i>	1320	1392	1440	1848

Source: Author

Table 2: Hurst coefficient for BTC/USD Volume

<i>BTC Volume</i>	12/27/2013 - 08/07/2017	12/27/2013 - 10/18/2017	12/27/2013 - 12/05/2017	12/27/2013 - 01/17/2019
<i>H</i>	0.273	0.262	0.28	0.309
<i>E(H)</i>	0.585	0.574	0.581	0.575
<i>t-stat</i>	-11.338	-11.627	-11.397	-11.415
<i>Standard Deviation</i>	0.028	0.027	0.026	0.023
<i>C</i>	-0.27	-0.281	-0.262	-0.232
<i>R-square</i>	0.986	0.986	0.991	0.993
<i>R-square E(H)</i>	0.997	0.998	0.998	0.998
<i>Number of observations</i>	1320	1392	1440	1848

Source: Author

Figure 2: R/S analysis and V-statistics of BTC/USD Close Prices and Volume, Data Period: 12/27/2013 – 08/07/2017 (1320 data points)

Source: Author's calculation

Table 3: Stability analysis of the Hurst coefficient of Bitcoin closing prices

	Up to date	H	E(H)	t-stat	R-square	R-square E(H)	type
	9/4/2015	0.5937	0.6714	-3.3391	0.9983	0.9994	persistent
	5/12/2017	0.5909	0.5703	0.8863	0.9970	0.9998	random
	1/17/2019	0.6504	0.5283	5.2466	0.9367	0.9999	persistent

Source: Author's calculation

Figure 2 to Figure 5 show the results of the R/S analysis, one for each analyzed period. We can see two lines, one for the R/S graph (black line) and the second one for the $E(R/S_n)$ graph (gray line). The $E(R/S_n)$ graph presents an independent process. We can see a systematic deviation from the expected values for all periods of Bitcoin closing price data. However, breaks in the R/S graph appear. We have calculated the V -statistics that is showed in Figure 2 to Figure 5 to estimate precisely where the breaks occur. Breaks occur when the V -statistics clearly stops growth. A break may be a signal of a periodic or non-periodic cycle in the analyzed time series. However, not every break represents another new cycle,

because it can be only a certain multiple of a smaller previous cycle. We have detected one significant 60-days-long cycle only during the period 12/27/2013 – 08/07/2017. The time series was persistent and significant. After that the time series stay random. It means that the further history has a long memory effect, the more recent history has a random character, and the Hurst coefficient is insignificant. We do not discover any cycle in the trading volume data during these periods. The Bitcoin price boom caused random behavior without any long-term memory dependence.

Figure 2: RS analysis and V-statistics of BTC/USD Close Prices and Volume, Data Period: 12/27/2013 – 10/18/2017 (1392 data points)

Source: Author's calculation

Figure 3: RS analysis and V-statistics of BTC/USD Close Prices and Volume, Data Period: 12/27/2013 – 12/05/2017 (1440 data points)

Source: Author's calculation

Table 4: Stability analysis of the Hurst coefficient of Bitcoin trading volume

	Up to date	H	E(H)	t-stat	R-square	R-square E(H)	type
	9/4/2015	0.3402	0.6713	-14.2358	0.9911	0.9994	Anti-persistent
	5/12/2017	0.2682	0.5703	-12.9870	0.9420	0.9998	Anti-persistent
	1/17/2019	0.3521	0.5283	-7.5769	0.9708	0.9999	Anti-persistent

Source: Author's calculation

Figure 4: RS analysis and V-statistics of BTC/USD Close Prices and Volume, Data Period: December 27, 2013 – January 17, 2019 (1848 data points)

A stability analysis of the estimation of the Hurst coefficient was realized for the whole data period. We divided the whole period into three equal sub-periods, and we estimated the Hurst coefficient. The result is given in Table 3 for Bitcoin closing prices. As we can see, during the period from 12/27/2013 to 09/04/2015, Bitcoin closing prices exhibit a persistent character that was statistically significant at the 0.05 confidence level. After that, until 05/12/2017, the closing prices show a random character. Finally, the last sub-period exhibits a persistent character, but with a higher value of the Hurst coefficient than it was during the first period. This result confirms the boom in the closing prices of Bitcoin.

Table 4 shows the results of the stability analysis for the traded volume. As we can see the Hurst coefficient showed an anti-persistent statistically significant character during all periods. Moreover, its values were comparable. We have found stationarity of the Hurst coefficient in trading volume during the whole analyzed period.

Table 5: Quantile regression results

quantile	Break date	08/07/17	p-value	10/18/17	p-value	12/05/17	p-value
0.05	$\Delta Close_{before}$	2128197	0.25	-592737	0.59	-1818547	0.02**
	$\Delta Close_{after}$	695593.5	0.00*	699264	0.00*	800687.5	0.00*
0.10	$\Delta Close_{before}$	1303699	0.22	64355.52	0.93	-1330892	0.01**
	$\Delta Close_{after}$	324755.1	0.02**	515101.7	0.00*	830205	0.00*
0.25	$\Delta Close_{before}$	153057	0.12	-102377	0.12	-658301	0.00*
	$\Delta Close_{after}$	489466.4	0.00*	488599.8	0.00*	734606.9	0.00*
0.50	$\Delta Close_{before}$	66186.26	0.00*	66036.19	0.00*	66186.26	0.00*
	$\Delta Close_{after}$	466770.8	0.00*	466731.6	0.00*	560616.2	0.00*
0.75	$\Delta Close_{before}$	166676.7	0.03**	297566.6	0.00*	528370.3	0.00*
	$\Delta Close_{after}$	447665.8	0.00*	446320.7	0.00*	347783.3	0.00*
0.90	$\Delta Close_{before}$	426167.9	0.68	730303.3	0.33	923820.9	0.08***
	$\Delta Close_{after}$	549841.7	0.00*	479035.2	0.00*	413480.5	0.01**
0.95	$\Delta Close_{before}$	267351.8	0.90	435689.2	0.75	435689.2	0.66
	$\Delta Close_{after}$	444937.5	0.11	439688.5	0.13	439688.5	0.15

* Sign=0.01, ** Sign=0.05, ***Sign=0.1

Source: Author

The last analysis describes a relation between changes of the closing prices and the traded volume of Bitcoin. Table 5 shows only slope coefficients of the quantile regression model (3). The break date is highlighted in the first row. We can see that after the break date the slope parameter was always positive and statistically significant with a growing tendency, mainly during bear market conditions (from quantile 0.05 to 0.25). The periods before the break date show either non-significant dependency, which

means changes in the closing prices did not affect the traded volume, or it had a negative impact on the traded volume. During normal market conditions (median) the slope parameters were always statistically significant and their values were comparable. When we take into account the 0.75 quantile, the impact of the changes of the closing prices on changes of the traded volume was statistically significant, but slightly lower than what we observed for the 0.25 quantile. The results for the 0.9 quantile were opposite to those of the 0.1 quantile. The market reacted by increasing the Bitcoin traded volume depending on the Bitcoin price increases. Finally, when Bitcoin prices neared their peak, investors stay indifferent to the Bitcoin prices.

Conclusion

In this paper, we have presented a time series fractal analysis of Bitcoin closing prices and Bitcoin traded volume. Only during further history, bitcoin closing prices showed a long memory effect, with the period being 60 days long. Bitcoin closing prices exhibited a random character after the boom in its prices (second half of the year 2017). The traded volume time series exhibit a stable anti-persistent character known as the mean reverting process. The volume was more volatile than a random series. The relationship between the changes of the traded volume and the changes of the closing prices was asymmetric. The price increase attracted investors and the growth of the volume was significant mainly during the bear market.

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