

EXPERIMENTAL STUDY OF DAMPED OSCILLATIONS OF A TORSIONAL PENDULUM

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Abstract: A laboratory work methodic on the experimental study of damping torsion oscillations under educational laboratory conditions was developed in this article. A torsion pendulum, which is usually used for bullet flight speed experimental determination based on angular momentum, mechanical energy conservation laws, and the laws of natural oscillations patterns, was used for this purpose. A torsion pendulum is a rod suspended on a vertically stretched steel wire and capable of performing oscillatory motion in the horizontal plane. Two rectangle shaped bodies are attached at the rod's ends, also two identical cylindrical weights that can be moved, changing the inertia moment of the pendulum, and locked in a chosen position, are arranged on the steel rod. Damping parameter dependencies are studied: the damping coefficient, damping logarithmic decrement, relaxation time, amount of oscillations during relaxation and the mechanical system quality factor against pendulum inertia moment. The least squares method was used in the processing of the experimental results. The results from this experimental study prove theoretical dependencies fidelity. The method used in this study may be recommended for physical practicum at universities when there is not a finished installation for the damping of torsional oscillations.

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Keyword: laboratory practicum, torsion pendulum, damping oscillations, damping characteristics, inertia moment.

Introduction

Physical laboratory practicum is an experimental study of physics, supported by modern educational equipment. Its main goal is to experimentally uncover the theoretical positions of physical science, to provide students with a deep understanding of the laws and forms of their development, to form practical handling skills objects of study, laboratory equipment and other experimental means, and to instill in them the skills of experimental activity. The conduction of laboratory works promotes the students' mastery of the principle of organic unity of theory and practice, their familiarization with the ways of developing experimental science, mastering them with the methods of experimental confirmation and verification of theoretical positions. The performance of laboratory works develops the interest of future specialists in the field of scientific research, and ensures the mastering of methods for planning and conducting experiments, of processing and analyzing the obtained data, and processing their results. These are important in the development of the professional qualities of future specialists and the skills of independent creative work.

Pendulums in laboratory practicum are used for different purposes. In the physical laboratory works by Evgrafova & Kagan (1970) and Korzun (1991), practical instructions of Syzdykova & Kenzhebekova (2015), and many other laboratory works, a physical reversible pendulum is used to study free mechanical oscillations and determine gravitational acceleration through Bessel's Method. In the article by Akimov, Baranov & Saletsky (2000), with the help of a training physical pendulum, ways to improve the accuracy of the measurements of gravitational acceleration were considered. Formulas for the calculation of errors of gravitational acceleration g are presented and a methodology to increase the accuracy of determination g in the conditions of the training laboratory was proposed.

In the guidelines by Syzdykova and Kenzhebekova (2007) were given guidelines for studying the free oscillations of a mathematical pendulum, by an experienced determination of the range of amplitudes in which a linear relationship between the square of the oscillation period and the length of the pendulum is performed and the period of oscillations remains constant with a specified accuracy. A formula to estimate the accuracy of the realization of the linear pendulum model in a laboratory facility was given. In the laboratory practicum by Korzun (1991) a spring-actuated pendulum is used to experimentally study natural oscillations, determine the stiffness coefficient of a spring, the oscillation period, and friction coefficient of a pendulum in a viscous medium. In the laboratory practicums of Astapov et al. (2011) and Nikulin and Grecev (2007), guidelines to study the damped oscillations of a physical pendulum and to determine their characteristics were given. In the article by Kachevskiy (2000), laboratory work is proposed for physical practicum where the parameters of free

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damped oscillations are studied with subsequent calculation of the material characteristics - the damping capacity and the shear modulus.

Pendulums were used not only for the experimental study of free vibrations and their characteristics, but also for the experimental determination of the inertial characteristics of a solid body, including the inertia moment of a pendulum and the experimental verification of Steiner's Theorem (Iskakov, 2014).

The torsion pendulum in laboratory practices of Avenarius & Afanasyev B.L. (2010) Salamatina, Karsybayev & Baipakbayev (2015) and many other practices were successfully used generally to determine bullet flight speed, to study solid bodies inertial characteristics, but we use it to study damping parameters dependences: damping coefficient, damping logarithmic decrement, relaxation time, the amount of oscillations during relaxation and the mechanical system quality factor against pendulum inertia moment and finally to determine material bending characteristics.

Experimental installation and methodic for the experimental study of damping characteristics depending on the moment of inertia of the torsion pendulum

In Figure 1, Polish manufacture FRM-09 torsion pendulum is a rod (1) suspended on a vertically stretched steel wire (2) and capable to perform oscillatory motion in the horizontal plane. Two rectangular shaped bodies are attached at the rod's ends along with two identical cylindrical weights (3) that can be moved and locked into a chosen position. From above, the pendulum is covered with a transparent cylindrical cap, the lateral side of which has a plotted angular scale (4). The measurement and control system consist of an electronic stopwatch and an oscillation counter, all are connected to a photoelectric sensor. POWER (5), RESET (6), STOP (7) - key switches and indicators, which show the stopwatch (8) and the oscillation counter (9) readings, are arranged on the front panel. Before the experiment, aligned the installation with foot screws (10), have established the pendulum against angular scale zero by turning the screw (11) and then fastened it in this position with the tension nuts (12). Manually have established loads (3) maximally nearer to each other by moving them along the pendulum axis. The installation was included in the electrical network.

Figure 1. Experimental installation



Source: Author

Now we cite the methodic for the experimental study of damping characteristics depending on the moment of inertia of the torsion pendulum. Turning the pendulum to angle A_0 equal to 20 angular scale divisions, providing that the pendulum position lock has closed the light path t to the photocell, pressing the RESET button, then releasing the pendulum were determined the times of $N = 10, 20, 30, 40, 50$ full oscillations and their corresponding oscillation amplitude values $A(N)$. Each experiment was repeated for not less than three times and were accepted the average values of their results.

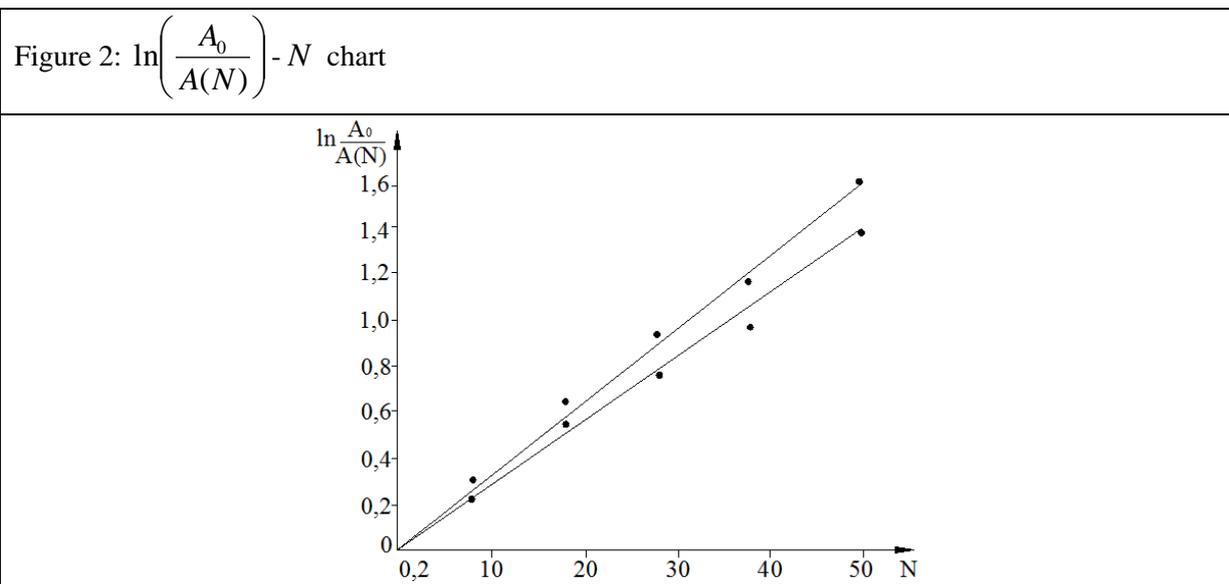
Calculation the oscillations period T according to the formula $T = \frac{t}{N}$ showed that the oscillations period values are practically identical in all measurements and, therefore, it was sufficient to take the value of time t , which corresponds for $N=30$ oscillations (in two positions of the cylindrical weights on the rod). For each number of oscillations, the value $\ln\left(\frac{A_0}{A(N)}\right)$ was calculated and the results of measurements and calculations were recorded in the table 1.

Table 1: Experimental Data

Cylindrical Weight Positions on the Rod	Oscillations Amplitude	Oscillations Amount N						t [s]
		0	10	20	30	40	50	
Maximally Approximated	A(N) grad	20	14	10	8	6	4	100,8
	$\ln\left(\frac{A_0}{A(N)}\right)$	0	0,3567	0,6931	0,9163	1,2040	1,6094	
Maximally Distant	A(N) grad	20	15	11	9	7	5	169,2
	$\ln\left(\frac{A_0}{A(N)}\right)$	0	0,2877	0,5978	0,7985	1,0500	1,3863	

Source: Author

Construct $\ln\left(\frac{A_0}{A(N)}\right) - N$ chart.



Source: Author

On the constructed line, some two points were taken and the damping logarithmic decrement was calculated according to the formula $\delta = tg\varphi$ and the number of oscillations during the relaxation time N_e corresponding to $\ln\left(\frac{A_0}{A(N)}\right) = 1$ by the formula $N_e = \frac{1}{\delta}$ was found. Further, according to the

formula $\beta = \frac{1}{N_e T} = \frac{\delta}{T}$, the attenuation coefficient, according to the formula $\tau = \frac{1}{\beta}$, the relaxation

time and finally, by the formula $Q = \frac{\pi}{\delta} = \pi N_e$ - the quality factor of the pendulum were calculated.

When processing the results of the experiment, the least squares method was used. (Syzdykova & Kenzhebekova, 2015; Diakonov, 1987). To do this, the following dependence $y = kx$ was adopted at

the following designations: $\ln\left(\frac{A_0}{A(N)}\right) = y, N = x$. Then $\delta = k = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$ and error is

$$\sigma_\delta = \sigma_k \approx \frac{1}{n} \sqrt{\frac{\sum_{i=1}^n y_i^2}{\sum_{i=1}^n x_i^2} - k^2}$$

Errors of other quantities were determined according to the expressions

$\sigma_{N_e} = \frac{1}{\delta^2} \sigma_\delta, \sigma_\beta = \frac{1}{T} \sigma_\delta + \frac{\delta}{T^2} \sigma_T, \sigma_\tau = \frac{1}{\delta} \sigma_T + \frac{T}{\delta^2} \sigma_\delta, \sigma_Q = \frac{\pi}{\delta^2} \sigma_\delta$. Further the previous stages of the experiment and calculations were repeated at the maximally distant positions of the cylindrical weights on the rod. The results of the calculations were introduced in Table 2.

Cylindrical Weights Positions on the Rod	T [s]	N_e	δ	β [s ⁻¹]	τ [s]	Q
Maximally Approximated	3,36	32	0,0316	0,0094	106	100
Maximally Distant	5,64	37	0,0273	0,0048	208	116

Source: Author

The two constructed $\ln\left(\frac{A_0}{A(N)}\right)$ - amount of oscillations N charts show that pendulum inertia moment I increase leads to the damping logarithmic decrement δ or oscillation coefficient β decrease, that shows the theoretical dependences fidelity of $\delta = \frac{r}{2I} T$ or $\beta = \frac{r}{2I}$, where r is the coefficient of resistance. Calculation results given in Table 2 also show the direct proportionality of relaxation time τ , amount of oscillations N_e in τ time, and mechanical system quality factor Q , against the pendulum inertia moment I .

From the textbook on the resistance of materials Darkov & Shpyro (1975), we have the expression for the angle of rotation of the pendulum $\alpha = \frac{Mab}{GI_p L}$, where a and b are the distances from the attachment point of the wire to the pendulum, $L = a + b$ - the total length of the wire, G - the shear modulus of the wire material, I_p - the polar moment of inertia of the wire transverse section, and M

- the torque. This formula can be represented in the form $\frac{GI_p L}{ab} \cdot \alpha = k \cdot \alpha = M$. The ratio of the coefficient at the torque to the moment of inertia of the pendulum is equal to the square of the natural frequency of the pendulum $\frac{k}{I} = \frac{GI_p L}{abI} = \omega_0^2 = \omega^2 + \beta^2$. Bearing in mind that, $\omega = \frac{2\pi}{T}$ and

$I_p = \frac{\pi d^4}{32}$, where d is the diameter of the wire cross-section, we find an expression for determining

the shear modulus of its material $G = \frac{32}{\pi d^4} \left(\frac{4\pi^2}{T^2} + \beta^2 \right) \frac{abI}{L}$. In the case of a symmetrical

arrangement of the pendulum with respect to the upper and lower attachment points of the wire, i.e.

when $a = b = L/2 = l$, the formula for G will take the following form $G = \frac{32}{\pi d^4} \left(\frac{4\pi^2}{T^2} + \beta^2 \right) Il$. If it

is possible to calculate or measure the moment of inertia of the pendulum, then the shear modulus of the wire material can be calculated from one of the formulas given. Thus, based on the conducted research, the following main conclusions can be presented.

Conclusion

The experimental installation, adapted for the study of damping oscillations under educational laboratory conditions, is represented in this article. A technique for laboratory work with a torsion pendulum, usually intended for bullet speed determining, was developed. The damping parameter dependencies were experimentally studied: damping coefficient, damping logarithmic decrement, relaxation time, the amount of oscillations during relaxation and the mechanical system quality factor against the pendulum inertia moment. Results were processed with a function approximation by the least squares method. The experimental results compared to the theoretical patterns shows their fidelity. The experimental installation and laboratory work technique can be recommended for physical practicum at universities as an effective and simple method to study damping torsional oscillation characteristics depending on the pendulum inertia moment.

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