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ON THE THEORY OF BENDING OF A THICK ORTHOTROPIC PLATE WITHOUT CONSIDERATION OF SIMPLIFYING HYPOTHESIS

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ABSTRACT

The problem of bending of a thick orthotropic plate is considered as a three-dimensional problem of the theory of elasticity. On the basic of the method of expansion of the solution into the series, a three-dimensional problem is reduced to two independent two –dimensional problems. The theory of thick orthotropic plates free from simplified hypothesis is developed: An analytical solution of equation is given. Maximum values of displacements and stresses for upper, middle and lower surfaces of the plate are calculated.

UDC CODE & KEYWORDS

■ UDC: 539.3 ■ Orthotropic Plate ■ Theory of Elasticity ■ Theory of Bending ■

INTRODUCTION

A great number of researchers have been studying the problem of the theory of bending of thick plates and shells, such as Timoshenko S.P., Reisner E., Galimov K.Z., Mushtari Kh.M., Vlasov B.F., Ambartsumyan A.S. and others. In Ambartsumyan's monograph [1] existing scientific data is divided into three groups. The third group of scientific works refers to the problems devoted to the reduction of three-dimensional problem of the theory of elasticity to two-dimensional one on the basis of the method of expansion of displacement into infinite series, without consideration of simplifying hypothesis. The problems of bending of thick plates were studied by many researchers as a three-dimensional problem of mechanics. In [2] an exact solution of spatial problem of bending of a thick orthotropic plate is built; Vlasov B.F. [3] has obtained an exact solution of the problem of bending of a thick isotropic plate in thigonometric functions. Such studies include the works of the authors: Vekua I.N. [4], Vlasov V.Z. [5], Galerkin B.G. [6], Kosmodamianskiy A.S., Shaldyrvan V.A. [7], Lurie A.I. [8] and others. In review of the monograph [9] scientific works devoted to this problem are given and systemized.

In spite of a wide-scale investigation of this problem, numeric results obtained, are not enough. This paper presents the method of expansion into series for the solution of spatial problem of bending of thick orthotropic plates. This paper presents a generalized version of published earlier works [10, 11] and is supplied with new theoretical materials and numeric results.

The statement of the problem

An orthotropic thick plate of constant thickness H=2 h is considered with dimensions a and b in plan. We will introduce Cartesian system of coordinates x_1, x_2 and z. An axis *Oz* is directed vertically downward. The plate experiences external distributed spatial normal forces q_3^+, q_3^- (along *Oz*) and tangential ones q_k^+, q_k^- ($k = \overline{1,2}$) (along Ox_k), applied to lower $z=h$ and upper $z=-h$ surfaces, respectively. The material of a plate obeys to Hook's generalized law [1]. Designation are introduced: E_1, E_2, E_3 - modulus of elasticity and G_{12}, G_{13}, G_{23} - modulus of shear, $\nu_{12}, \nu_{13}, \nu_{23}$ - Poison's coefficients; σ_{ij} and ε_{ij} ($i, j = \overline{1,3}$) – elements of the tensor of stress and strain, u_1, u_2, u_3 - components of displacement vector.

Further we will use designations and laws of the theory of elasticity in relation of indexes *i* and *j* in one term.

Thick plate is considered as a spatial body. The equilibrium equation of three-dimensional theory of elasticity is written down in the following form

$$\sigma_{kj,j} = 0, \qquad (k, j = 1,3) \qquad (1)$$

here *j* is taken as a summing up.

Boundary conditions on a lower z= h and upper z=- h surfaces are given in the form:

$$\sigma_{33} = q_3^+, \quad \sigma_{3k} = q_k^+, \quad \text{at} \quad z = h; \qquad (2)$$

$$\sigma_{33} = q_3^-, \quad \sigma_{3k} = q_k^-, \quad \text{at} \quad z = -h. \qquad (3)$$

Boundary conditions on lateral faces of a plate are given on a lower level depending on the conditions of fixing of a plate.

The solution of the problem of bending of thick plates is built on the basis of the method of expansion in degrees of the relationship *z*/*h* along the thickness of a plate, that is displacements are presented in the form of Macloren's series:

$$u_k = B_0^{(k)} + B_1^{(k)} \frac{z}{h} + B_2^{(k)} \left(\frac{z}{h}\right)^2 + B_3^{(k)} \left(\frac{z}{h}\right)^3 + \cdots, (k = 1,2),$$

$$u_3 = A_0 + A_1 \frac{z}{h} + A_2 \left(\frac{z}{h}\right)^2 + A_3 \left(\frac{z}{h}\right)^3 + \cdots. \qquad (4)$$

Here $B_i^{(k)}$, A_i - are unknown functions of two spatial coordinates:

$$B_i^{(k)} = B_i^{(k)}(x_1, x_2), \quad A_i = A_i(x_1, x_2)$$

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Displacements, presented in the form of series (4), must satisfy the equations of the theory of elasticity (1) and boundary conditions (2).

In [10] two separate systems of equations are built; the first one is described in relation to unknown coefficients of the series (4) $B_0^{(1)}, B_0^{(2)}, A_1$, and the second one – in relation to unknown coefficients of the series (4) $B_1^{(1)}, B_1^{(2)}, A_0$. On the basis of analysis of numeric results it was stated that to obtain numeric results of satisfying accuracy it is necessary to take into account eight terms in every series (4) as a minimum. This leads to two-dimensional theory with differential equations of a higher order. To eliminate this disadvantage in [11] a new two-dimensional theory of thick plates without consideration of simplifying hypothesis was offered. Here we will briefly state the procedure of building of these equations.

To the theory of bending of thick plates

Displacements of upper fibers (z=-h) of the plate we will designate as $u_k^{(-)}, (k=1,3)$, and displacements of lower fibers (z=h) - $u_k^{(+)}, (k=1,3)$. We obtain formulae for the determination of displacements for upper fibers (z=-h) of the plate $u_k^{(-)}, (k=1,3)$ in the form:

$$u_k^{(-)} = \overline{U}_k - \widetilde{U}_k, \quad u_3^{(-)} = \widetilde{W} - \overline{W} \qquad (5)$$

To calculate displacements of lower fibers (z=h) $u_k^{(+)}, (k=1,3)$, the following expressions were obtained

$$u_k^{(+)} = \overline{U}_k + \widetilde{U}_k, \quad u_3^{(+)} = \widetilde{W} + \overline{W}, \qquad (6)$$

Where

$$\widetilde{U}_k = B_1^k + B_3^k + ... + B_{2n+1}^k \ , \ \widetilde{W} = A_0 + A_2 + ... + A_{2n}, \ (7)$$

$$\overline{U}_k = B_0^k + B_2^k + ... + B_{2n}^k, \ \overline{W} = A_1 + A_3 + ... + A_{2n+1}. \ (8)$$

So, the connection between the values of displacements $u_j (j=1,3)$ and coefficients of expansion $B_i^{(k)} (k=1,2)$ and A_i was stated. Hence one may draw a conclusion that if coefficients of expansion into series (4) satisfy conditions (7) and (8), expansion (4), converge because the series (7) and (8) are majorants of the series (4).

To build two-dimensional equations we will introduce two-dimensional operators of differentiation of the second order:

$$L_1(f_1, f_2) = E_{11}\frac{\partial^2 f_1}{\partial x_1} + (E_{12} + G_{12})\frac{\partial^2 f_2}{\partial x_1 \partial x_2} + G_{12}\frac{\partial^2 f_1}{\partial x_2^2},$$

$$L_2(f_1, f_2) = G_{21}\frac{\partial^2 f_2}{\partial x_1^2} + (E_{12} + G_{21})\frac{\partial^2 f_1}{\partial x_1 \partial x_2} + E_{22}\frac{\partial^2 f_2}{\partial x_2^2},$$

$$L_3(f) = G_{31}\frac{\partial^2 f}{\partial x_1^2} + G_{32}\frac{\partial^2 f}{\partial x_2^2}{}_3 \ .$$

For convenience we will introduce the designations

$$\overline{q}_k = \frac{q_k^+ - q_k^-}{2}, \quad \overline{q}_3 = \frac{q_3^+ + q_3^-}{2}, \quad \widetilde{q}_k = \frac{q_k^+ + q_k^-}{2}, \quad \widetilde{q}_3 = \frac{q_3^+ - q_3^-}{2}.$$

It should be stated that in general case tensile and cutting forces, bending and torsional moments are determined through twelve unknown functions, which make two separate systems of equations with three equations in each system, having six unknown functions. The first system consists of two equations of forces, acting in the plane of a plate and is obtained by integration of two first equations of the theory of elasticity according to z coordinate:

$$\frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{12}}{\partial x_2} = -2\overline{q}_1, \frac{\partial N_{21}}{\partial x_1} + \frac{\partial N_{22}}{\partial x_2} = -2\overline{q}_2 \ (9a)$$

And the third equation is obtained by multiplying the third equation of the theory of elasticity on z coordinate and integrating according to z

$$\overline{p}_{33} + H\frac{\partial p_{13}}{\partial x_1} + H\frac{\partial p_{23}}{\partial x_2} = H\overline{q}_3 \qquad (9b)$$

The system of equations (9) includes six unknown functions $\overline{U}_1, \overline{U}_2$, $\overline{W}$, $\overline{F}_1, \overline{F}_2, \overline{r}$ and is written down in the following form:

(10)

here m =1,2,3,…; $\alpha_{3k} = \frac{G_{3k}}{G_{3k} + E_{3k}}$; $\alpha_{3k} = \alpha_{k3}$

The system of equation (10) except for unknowns $\overline{W}$ and $\overline{U}_k, (k=1,2)$, determined by correlations (7) and (8), includes unknown functions, determined by coefficients of the series A_{2j+1} and $B_{2j}^{(k)}$ (4) in the form

$$\overline{F}_k = -\frac{4}{3}B_2^{(k)} - \frac{8}{5}B_4^{(k)} - \frac{12}{7}B_6^{(k)} - ... - \frac{4n}{2n+1}B_{2n}^{(k)} + ..., \ (11)$$

$$\overline{r} = \frac{1}{3}A_1 + \frac{1}{5}A_3 + \frac{1}{7}A_5 + ... + \frac{1}{2n+1}A_{2n-1} + ... + \qquad (12)$$

The second system of equations includes two equations relative to bending and torsional moments and one equation of cutting forces

$$\frac{\partial M_{11}}{\partial x_1} + \frac{\partial M_{12}}{\partial x_2} - Q_{13} = -2\widetilde{q}_1 \ , \ \frac{\partial M_{21}}{\partial x_1} + \frac{\partial M_{22}}{\partial x_2} - Q_{23} = -2\widetilde{q}_2 \ (13a)$$

$$\widetilde{\sigma}_{33} + H\frac{\partial Q_{13}}{\partial x_1} + H\frac{\partial Q_{23}}{\partial x_2} = H\widetilde{q}_3 \ (13b)$$

They are written in displacements in the form:

$$\begin{cases} H^2 L_k(\tilde{\psi}_1,\tilde{\psi}_2) - \dfrac{H}{\alpha_{k3}} G_{3k} \dfrac{\partial \tilde{F}}{\partial x_k} - G_{3k}\left(4\tilde{U}_k + 2H\dfrac{\partial \tilde{W}}{\partial x_k}\right) = -2H\tilde{q}_k\,, \quad (k=1,2); \\ \dfrac{H^2}{2} L_3(\tilde{F}) + H^2 L_3(\tilde{W}) + 2G_{3i} H \dfrac{\partial \tilde{U}_i}{\partial x_i} = -2H\tilde{q}_3\,, \quad (i=1,2). \end{cases} \quad (14)$$

In the system of equations (14) except unknown $\tilde{W}$ and $\tilde{U}_k$, $(k=1,2)$, determined by relationships (7) and (8), there are unknown functions, determined by coefficients of the series A_{2j} and $B_{2j+1}^{(k)}$ (4) in the form:

$$\tilde{\psi}_k = \frac{1}{3}B_1^{(k)} + \frac{1}{5}B_3^{(k)} + \frac{1}{7}B_5^{(k)} + \ldots + \frac{1}{2n+1}B_{2n-1}^{(k)} + \ldots,$$

(k=1,2) (15)

$$\tilde{F} = -\frac{4}{3}A_2 - \frac{8}{5}A_4 - \frac{12}{7}A_6 - \ldots - \frac{4n}{2n+1}A_{2n} + \ldots \quad (16)$$

Here E_{ij} - are elastic constants, connected with modulus of elasticity and Poisson's coefficients:

$$E_{11} = E_1 * g_{11}\,,\quad E_{22} = E_2 * g_{22}\,,\quad E_{33} = E_3 * g_{33}$$

$$E_{12} = E_{21} = E_1 * g_{12} = E_{12} * g_{21}\,,\quad E_{13} = E_{31} = E_1 * g_{13} = E_3 * g_{31}$$

$$E_{23} = E_{32} = E_2 * g_{23} = E_3 * g_{32}$$

$$g_{11} = \frac{1-\nu_{23}\nu_{32}}{1-\mu^2}\,,\quad g_{12} = g_{21} = \frac{\nu_{12}+\nu_{13}\nu_{32}}{1-\mu^2} = \frac{\nu_{21}+\nu_{31}\nu_{23}}{1-\mu^2}$$

$$g_{22} = \frac{1-\nu_{13}\nu_{31}}{1-\mu^2}\,,\quad g_{13} = g_{31} = \frac{\nu_{13}+\nu_{21}\nu_{32}}{1-\mu^2} = \frac{\nu_{31}+\nu_{12}\nu_{23}}{1-\mu^2}$$

$$g_{22} = \frac{1-\nu_{12}\nu_{21}}{1-\mu^2}\,,\quad g_{23} = g_{32} = \frac{\nu_{23}+\nu_{13}\nu_{12}}{1-\mu^2} = \frac{\nu_{32}+\nu_{31}\nu_{21}}{1-\mu^2}$$

So, two separate systems of differential equations of equilibrium were built, with three equations in each one. To build other lacking equations of equilibrium of thick plates we will use recurrence equations (1), boundary conditions (2), designations (7), (8) and series (4). On the basis of these relationships, using equations of the theory of elasticity (1), we will build two systems of linear algebraic equations in relation to coefficients of series (4). As a right free terms (known ones) in the first system there are unknown functions $\bar{U}_1,\bar{U}_2$, $\bar{W}$, $\bar{F}_1,\bar{F}_2,\bar{r}$ and their partial derivatives, and in the second system - unknown $\tilde{U}_1,\tilde{U}_2,\tilde{W}$, $\tilde{\psi}_1,\tilde{\psi}_2,\tilde{F}$ and their partial derivatives. Solving these systems of algebraic equations, we determine the coefficients of the series (4) through unknown functions and their partial derivatives. Confined by six terms of expansion (4) we obtain two more separate differential equations. The first system of equations of equilibrium has the form:

$$\frac{\partial \bar{\sigma}_{11}}{\partial x_1} + \frac{\partial \bar{\sigma}_{12}}{\partial x_2} - p_{13} = -2\bar{q}_1\,,\quad \frac{\partial \bar{\sigma}_{21}}{\partial x_1} + \frac{\partial \bar{\sigma}_{22}}{\partial x_2} - p_{23} = -2\bar{q}_2 \quad (17a)$$

$$\bar{\sigma}_{33} + H\frac{\partial \bar{\sigma}_{13}}{\partial x_1} + H\frac{\partial \bar{\sigma}_{23}}{\partial x_2} = H\bar{q}_3 \quad (17b)$$

The system of equations in displacements is rewritten in the form (18a)

$$G_{3k}\bar{F}_k = \frac{2}{5}G_{3k}\bar{\varphi}_k - \frac{1}{30}\left(H^2 L_k(\bar{U}_1,\bar{U}_2) - \frac{H^2}{\alpha_{k3}}G_{3k}\frac{\partial}{\partial x_k}\left(\frac{E_{3i}}{E_{33}}\frac{\partial \bar{u}_i}{\partial x_i} - \frac{\bar{q}_3}{E_{33}}\right)\right),\ (k=1,2)$$

$$E_{33}\bar{r} = E_{33}\frac{2}{5}\bar{W} + \frac{1}{30}(E_{3i}H\frac{\partial \bar{u}_i}{\partial x_i} - H\bar{q}_3) \quad (18b)$$

The systems of equations (10) and (18) compose a closed system of equations in relation to unknown functions $\bar{U}_1,\bar{U}_2$, $\bar{W}$, $\bar{F}_1,\bar{F}_2,\bar{r}$

The second system of equations is written in the form:

$$\tilde{p}_{k3} = \tilde{q}_{k3}, (k=1,2) \quad (19a)$$

$$\tilde{p}_{33} + H\frac{\partial \tilde{p}_{13}}{\partial x_1} + H\frac{\partial \tilde{p}_{23}}{\partial x_2} = H\tilde{q}_3 \quad (19b)$$

The system of equations in displacements is rewritten in the form

$$G_{3k}\tilde{\psi}_k = G_{3k}\left(\frac{2}{5}\tilde{U}_k + \frac{1}{30}H\frac{\partial W}{\partial x_k}\right) - \frac{H}{30}q_k\,,\ (k=1,2) \quad (20a)$$

$$E_{33}\tilde{F} = \frac{2}{5}E_{3i}H\frac{\partial \tilde{U}_i}{\partial x_i} + \frac{1}{30}H^2E_{3i}\frac{\partial^2 W_i}{\partial x_i^2} - \frac{2}{5}H\tilde{q}_3 - \frac{1}{30}H^2\frac{\partial \tilde{q}_i}{\partial x_i} \quad (20b)$$

Systems (14) and (20) – compose joint system of equations in relation to unknown functions $\tilde{U}_1,\tilde{U}_2,\tilde{W},\quad \tilde{\psi}_1,\tilde{\psi}_2,\tilde{F}$

Similar formulae were obtained for the determination of maximum values of stresses on upper fiber (z=-h)

$$\sigma_{11}^{(-)} = \bar{\sigma}_{11} - \tilde{\sigma}_{11},\quad \sigma_{12}^{(-)} = \bar{\sigma}_{12} - \tilde{\sigma}_{12},\quad \sigma_{22}^{(-)} = \bar{\sigma}_{22} - \tilde{\sigma}_{22} \quad (21a)$$

and lower fiber (z=h) of a plate

$$\sigma_{11}^{(+)} = \bar{\sigma}_{11} + \tilde{\sigma}_{11},\quad \sigma_{12}^{(+)} = \bar{\sigma}_{12} + \tilde{\sigma}_{12},\quad \sigma_{22}^{(+)} = \bar{\sigma}_{12} + \tilde{\sigma}_{22}. \quad (21b)$$

Here the following designations were taken $\bar{\sigma}_{11},\bar{\sigma}_{12},\bar{\sigma}_{22}$ - the values of stresses, determined while solving the problems (A):

$$\bar{\sigma}_{11} = \left(E_{1i} - \frac{E_{13}}{E_{33}}E_{3i}\right)\frac{\partial \bar{U}_i}{\partial x_i} + \frac{E_{13}}{E_{33}}\bar{q}_3,\quad \bar{\sigma}_{22} = \left(E_{2i} - \frac{E_{23}}{E_{33}}E_{3i}\right)\frac{\partial \bar{U}_i}{\partial x_i} + \frac{E_{23}}{E_{33}}\bar{q}_3,$$

$$\bar{\sigma}_{12} = G_{12}\left(\frac{\partial \bar{U}_i}{\partial x_2} + \frac{\partial \bar{U}_2}{\partial x_i}\right) \quad (22)$$

Similarly, $\tilde{\sigma}_{11},\tilde{\sigma}_{12},\tilde{\sigma}_{22}$ - the values of stresses, while solving the problem (B), are determined with the following formulas:

$$\tilde{\sigma}_{11} = \left(E_{1i} - \frac{E_{13}}{E_{33}}E_{3i}\right)\frac{\partial \tilde{U}_i}{\partial x_i} + \frac{E_{13}}{E_{33}}\tilde{q}_3,\quad \tilde{\sigma}_{22} = \left(E_{2i} - \frac{E_{23}}{E_{33}}E_{3i}\right)\frac{\partial \tilde{U}_i}{\partial x_i} + \frac{E_{23}}{E_{33}}\tilde{q}_3,$$

$$\tilde{\sigma}_{12} = G_{12}\left(\frac{\partial \tilde{U}_i}{\partial x_2} + \frac{\partial \tilde{U}_2}{\partial x_i}\right). \quad (23)$$

Below we consider numeric example.

Solution of the problem of transversal bending of the plate.

Consider transversal bending of thick plate with boundary conditions of hinged fastening on lateral surfaces:

$$\sigma_{11}=0,\ u_2=0,\ u_3=0\ \text{, (at } x_1=0;\ \ a\ \text{)},$$

$$\sigma_{22}=0,\ u_1=0,\ u_3=0\ \text{, (at } x_2=0;\ \ b\ \text{). (24)}$$

Let the thick plate on upper surface z=-h is subjected to the action of normal distributed force only,

$$q_k^+=q_k^-=0,(k=1,2);$$

$$q_3^-=-q\sin\frac{\pi x_1}{a}\sin\frac{\pi x_2}{b},$$

where q – is a parameter of load.

Solution of the system of equation (10) and (18), satisfying boundary conditions (24) has the form

$$\overline{U}_1=c_1\cos(\frac{\pi x}{a})\sin(\frac{\pi y}{b}),\quad \overline{U}_2=c_2\cos(\frac{\pi y}{b})\sin(\frac{\pi x}{a}),$$

$$\overline{W}=c_3\sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b}).\qquad (25)$$

Solution of the system of equations (14) and (20) satisfying boundary conditions (24) has the form

$$\tilde{U}_1=c_4\cos(\frac{\pi y}{b})\sin(\frac{\pi x}{a}),\quad \tilde{U}_2=c_5\sin(\frac{\pi y}{b})\cos(\frac{\pi x}{a}),$$

$$\tilde{W}=c_6\sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b}).\qquad (26)$$

For concrete parameters of a plate calculations of displacements and stresses were carried out. Calculations were carried out for square (rectangular) *(a=b)* isotropic plate with relative thickness H/a =1/3 and Poisson's coefficient ν =0,3 and $G_{12}/G_{13}=1$. Based on calculations, given in Tables 1-4, one may draw a conclusion that displacements of upper and lower layer of a plate differ considerably from each other. The difference is greater in normal displacement u_3 (bending). Displacements u_1 , u_2 and stresses σ_{11}, σ_{12} on middle surface (z=0) are negligibly small (Tables 1 and 2).

Table 1: Maximum values of dimensionless displacements of isotropic plate

	Eu₁/Hq	***Eu₂/Hq***	***Eu₃/Hq***
z=-h	1,1356	1,1356	3,6012
z=0	-0,1095	-0,1095	3,4961
z=h	-1,2483	-1,2483	3,149

Maximum value of stresses was discovered on upper surface z=-h σ_{11}/q =2,1270 (Table 2), which considerably differs from the value of stresses of lower layer z=h σ_{11}/q =1,8675 (Table 4); this value is a compressive one.

So the hypothesis of un-compressible character of a plate in transversal direction and the hypothesis of plane sections are not proved.

Table 2: Maximum values of dimensionless stresses of isotropic plate

	s₁₁/q	***s₁₂/q***	***s₂₂/q***
z=-h	-2,1274	-0,9147	-2,1274
z=0	-0,0478	-0,0882	-0,0478
z=h	1,8675	-0,10056	1,8675

Table 3 presents the values of dimensionless stresses $\sigma_{33},\sigma_{13},\sigma_{23}$ for three values of relative thickness H/*a*.

Comparing numeric results obtained, it was stated that numeric results, given in Tables 1 - 4 and results of exact calculation [3] coincide with high accuracy both in displacements and stresses. It should be noted that displacements and stresses of upper and lower layers were calculated according to formulae (5) – (8), (21) – (23).

Table 3: Maximum values of dimensionless stresses of isotropic plate

	H/a	***s₃₃/q***	***t₁₃/q***	***t₂₃/q***
z=0	1.10	-0,494	2,3715	2,3715
	1.5	-0,5032	1,2623	1,1623
	1.3	-0,5083	0,7024	0,7024

Tables 4 - 5 show maximum values of dimensionless displacements and stresses, obtained while solving the problem of bending of isotropic plate by relative geometric characteristics $H/a=1/5$, $H/b=1/3$. Calculations show (Table 5), that normal stress σ_{11} , directed along the plate of greater size is less than the stress σ_{22} , directed along the plate of small size.

Table 4: Maximum value of dimensionless displacements of isotropic plate

	Eu₁/Hq	***Eu₂/Hq***	***Eu₃/Hq***
z=-h	1,4829	2,4716	6,7103
z=0	-0,0926	-0,0746	6,7291
z=h	-1,6022	-2,6704	6,2557

Table 5: Maximum value of dimensionless stresses of isotropic plate

	s₁₁/q	***s₁₂/q***	***s₂₂/q***
z=-h	-2,3057	1,1946	-3,5799
z=0	-0,0964	-0,0882	-0,0673
z=h	2,0282	-1,2906	3,4049

Tables 6 and 7 give results of calculation for orthotropic plate by relationships of elastic characteristics $E_1/G_{13}=4$, $E_2/G_{13}=3$, $E_3/G_{13}=2$, $G_{12}/G_{13}=1{,}5$, $G_{23}=G_{13}$, Poisson's coefficients $\nu_{12}=0{,}25$, $\nu_{13}=\nu_{23}=0{,}3$ and relative geometric characteristics $H/b=1/5$, $H/a=1/3$.

Table 6: Maximum values of dimensionless displacements of orthotropic plate

	Eu_1/Hq	Eu_2/Hq	Eu_3/Hq
z=-h	2,5942	1,5869	8,1289
z=0	-0,3047	-0,2298	8,264
z=h	-3,0021	-1,9239	7,2759

Table 7: Maximum values of dimensionless stresses of orthotropic plate

	s_{11}/q	s_{12}/q	s_{22}/q
z=-h	-2,7438	1,2344	-1,9889
z=0	-0,0509	-0,162	-0,0325
z=h	3,4092	-1,4629	1,4345

CONCLUSION

Based on calculations, given in the tables, one may draw a conclusion that displacements of upper and lower layer of a plate differ considerably from each other. The difference is greater in normal displacement. Displacements and stresses on middle surface are negligibly small. Maximum value of stresses was discovered on upper surface, which considerably differs from the value of stresses of lower layer, this value is a compressive one. So, the hypothesis of un-compressible character of a plate in transversal direction and the hypothesis of plane sections are not proved. By this way, created a theory of thick ortotropic plates without simplifying hypothesis and solved the problem of plate bending on the basis of this theory.

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